參考用

1. **(20 points)** Let F be a field, $n \geq 2$ be an integer, and let V be the vector space of $n \times n$ matrices over F. Let A be a fixed element of V and let T be a linear operator on V defined by $T(B) = AB$.
(a) Prove that T and A have the same minimal polynomial.
(b) If A is diagonalizable, prove, or disprove by counterexample, that T is diagonalizable. (10 pts)
(c) Do A and T have the same characteristic polynomial? Why or why not? (5pts)

2. **(20 points)** Let V be a finite dimensional vector space over $\mathcal{R}$ and f and g two linear functionals on V. If $\ker f = \ker g$ show that g is a scalar multiple of f.

3. **(20 points)** Let A and B be $n \times n$ matrices with entries from $\mathcal{R}$.
(a) Prove that if one of the two matrices, say A, is invertible then AB and BA are similar. (5pts)
(b) Show that there exist matrices A and B so that AB is *not* similar to BA. (5pts)
(c) Prove that the *characteristic polynomials* of AB and BA are the same.

4. **(20 points)** Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.
(a) Find the Jordan canonical form of A. (10pts)
(b) Find a matrix T (with entries in C) such that $T^{-1}AT$ is diagonal, or prove that such a matrix does not exist.

5. **(20 points)** Find a matrix whose minimal polynomial is $x^2(x-1)^2$, whose characteristic polynomial is $x^4(x-1)^3$ and whose rank is 5.