國立中央大學109學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生) 共1頁 第1頁
統計研究所 碩士班 不分組(在職生)

科目： 數理統計 ＊計算題需計算過程，無計算過程者不予計分

本科考試可使用計算器，廠牌、功能不拘 ＊請在答案卷(卡)內作答

1. A die is rolled, with equal probability for each face to turn up. Let X be the face value that turns up; that is, $P(\{X=i\})=\frac{1}{6}$, for $i=1,2,\dots,6$. Let $Y=X_1+X_2$, where X_1 and X_2 are two independent random variables following this distribution. Compute the *cumulative distribution function* of Y. (10%)

2. Let $X_1,X_2,\dots,X_n$ be sampled from a continuous distribution with finite mean μ $(\mu\neq 0)$ and finite variance σ^2. Suppose one wants to generate random variables that follow the distribution of the harmonic mean $\frac{n}{\sum_{i=1}^{n} X_i}$ under large sample size.

   (a) Find the asymptotic distribution of $\frac{n}{\sum_{i=1}^{n} X_i}$ as $n\to\infty$. (15%)

   (b) Prove that for any continuous random variable X with the cumulative distribution function $F(\cdot)$, $F(X)$ has the *Uniform* (0,1) distribution. (10%)

   (c) Based on (b), provide a procedure of generating random variables that follow the asymptotic distribution in (a). (10%)

3. A coin is tossed twice. Let X denote the number of heads $(X\in\{0,1,2\})$ and be modeled by the *Binomial* $(2,\theta)$, where θ is the probability of heads in a single toss. For this coin, it is known from previous experiments that $\theta\in\{\frac{1}{2},\frac{1}{3}\}$. Given this condition, compute the maximum likelihood estimate of θ if $X=0$ is observed. (10%)

4. Let $X_1,X_2,\dots,X_n$ be a random sample from the density function $f(x;\alpha,\beta)=\frac{1}{\beta}\exp\{-\frac{x-\alpha}{\beta}\}$, where $\alpha<x<\infty$, $-\infty<\alpha<\infty$, and $\beta>0$.

   (a) Find the maximum likelihood estimators of (α,β). (10%)

   (b) Assume $\alpha<1$. Find the maximum likelihood estimator of $P(\{X_1\geq 1\})$. (10%)

5. Let $X_1,X_2,\dots,X_n$ be a random sample from *Poisson distribution* with mean θ. Find the uniformly minimum variance unbiased estimator (UMVUE) of $P(\{X_1=k\})$, where k is a fixed positive integer. (10%)

6. Find a most powerful test of size α for $H_0\colon X\sim f_0(x)$ against $H_1\colon X\sim f_1(x)$ based on a sample of size one, where $f_0(x)$ is the density function of the *standard normal distribution* and $f_1(x)=\frac{1}{2}\exp\{-|x|\}$, $x\in\mathbb{R}$. (15%)

參考用