國立中央大學114學年度碩士班考試入學試題

系所： 統計研究所碩士班 不分組(一般生)
統計研究所碩士班 不分組(在職生)

科目： 數理統計

＊本科考試可使用計算器，廠牌、功能不拘

計算題應詳列計算過程，無計算過程者不予計分

Statistics

1. Let $f(x,y) = e^{-y}$ for $0 < x < y < \infty$ be the joint probability density function of the random variables X and Y.

   (a) (10%) Find the marginal probability density function $f(y)$ and the conditional probability density function $f(y|x)$.

   (b) (8%) Prove that $\mathrm{Var}(Y|X = x) \le \mathrm{Var}(Y)$ and offer an intuitive interpretation of this result.

2. Let X and Y be two non-negative random variables such that $X|Y \sim$ Poisson(Y) and $Y \sim$ Exponential(θ), where $\theta > 0$.

   (a) (10%) Find the marginal probability mass function of X.

   (b) (6%) Find the expected value of X.

3. Let $X_1, \ldots, X_n$ be a random sample from the uniform distribution $U(0,\theta)$ with $\theta > 0$ and let Y_n be the n-th order statistic of $\{X_1, \ldots, X_n\}$.

   (a) (6%) Find the probability density function of Y_n.

   (b) (8%) Find the maximum likelihood estimator of $\mathrm{E}(Y_n)$.

   (c) (8%) Show that Y_n is a consistent estimator of θ as $n \longrightarrow \infty$.

4. Let $X_1, \ldots, X_n$ be a random sample from the Bernoulli distribution Ber(p) with $p \in (0,1)$.

   (a) (6%) Find a complete sufficient statistic for p.

   (b) (8%) Show that the uniformly minimum variance unbiased estimator of $p(1-p)$ is $n\bar{X}(1-\bar{X})/(n-1)$.

   (c) (8%) Obtain the Bayes estimator of $p(1-p)$ under the squared error loss with respect to the Beta prior distribution Beta(α, β), where α and β are known.

注意:背面有試題

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5. Let $X_1, \ldots, X_n$ be a random sample from a distribution with the probability density function $f(x;\theta) = 2x/\theta^2$, where $0 < x < \theta$ and $\theta > 0$.

   (a) (6%) Find a sufficient statistic for θ.

   (b) (8%) Determine the maximum likelihood estimator of θ.

   (c) (8%) Construct a $1-\alpha$ confidence interval for θ.

Supplementary materials:

- **Bernoulli**(p): $P(X = x|p) = p^x(1-p)^{1-x}$; $x = 0, 1$; $0 \le p \le 1$
- **Poisson**(λ): $P(X = x|\lambda) = e^{-\lambda}\lambda^x/x!$; $x = 0, 1, \ldots$; $0 \le \lambda < \infty$
- **Uniform**(a, b): $f(x|a, b) = \frac{1}{b-a}$; $a \le x \le b$
- **Beta**(α, β): $f(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$; $0 \le x \le 1$; $\alpha > 0$; $\beta > 0$
- **Gamma**(α, β): $f(x|\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$; $0 \le x < \infty$; $\alpha > 0$; $\beta > 0$
- **Exponential**(β): $f(x|\beta) = \frac{1}{\beta}e^{-x/\beta}$; $0 \le x < \infty$; $\beta > 0$

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