國立中央大學八十四學年度碩士班研究生入學試題卷

所別：統計研究所 甲組 科目：數理統計 共 / 頁 第 / 頁

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from a population with p.d.f. $f(x|\theta) = \theta x^{\theta-1}, 0 < x < 1$ and $\theta > 0$.

   (a) Find the sufficient statistic of θ. (5%)

   (b) Find the method of moment estimator of θ. Is the estimator sufficient ? Why ? (8%)

   (c) Find the maximum likelihood estimator (MLE) of θ. Is the MLE sufficient ? Why ? (8%)

   (d) Find the MLE of the mean of the distribution. (5%)

2. Let $X_1, X_2, \cdots, X_n$ be a random sample from a population with p.d.f. $f(x|\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, x \geq 0$ and $\theta > 0$.

   (a) Find the uniformly minimum variance unbiased estimator (UMVUE) of θ. (8%)

   (b) Find the UMVUE of $g(\theta) = \int_0^t [1 - F(x|\theta)]dx = \int_0^t e^{-\frac{x}{\theta}}dx = \theta(1 - e^{-\frac{t}{\theta}})$ for $t > 0$. (12%)

3. Let $X_1, X_2, \cdots, X_n$ be a random sample from the population with pmf $f(x|\theta) = \frac{e^{-\theta}\theta^x}{x!}$, $x = 0, 1, 2, \cdots$ and $\theta > 0$.

   (a) Find the MLE $\hat{\theta}$ of θ. (5%)

   (b) Find an approximate 95% confidence interval (C.I.) for θ by using the asymptotic distribution of $\hat{\theta}$. (8%)

   (c) Find an approximate 95% C.I. for $\frac{1}{\theta}$. (5%)

4. Let θ have 3 possible values θ_1, θ_2 and θ_3, and the probability distributions corresponding to different values of θ are given below.

i	0	1	2	3
$p(i\|\theta_1)$	0.05	0.05	0.10	0.80
$p(i\|\theta_2)$	0.05	0.80	0.15	0
$p(i\|\theta_3)$	0.90	0.08	0.02	0

   A single random variable X is observed.

   (a) Find the MLE of θ. (5%)

   (b) Find the likelihood-ratio test for testing $H_0 : \theta = \theta_1$ v.s. $H_1 : \theta \in \{\theta_2, \theta_3\}$, $\alpha = 0.10$. (8%)

   (c) Is there a uniformly most powerful test for testing $H_0 : \theta = \theta_1$ v.s. $H_1 : \theta \in \{\theta_2, \theta_3\}$, $\alpha = 0.10$? Justify your answer. (5%)

5. Let X_1 and X_2 be a random sample from a population with the uniform $(\theta, \theta + 1)$ distribution. For testing $H_0 : \theta = 0$ v.s. $H_1 : \theta > 0$, we have two competing tests:
   (i) $\phi_1(X_1)$: Reject H_0 if $X_1 > 0.95$
   (ii) $\phi_2(X_1, X_2)$: Reject H_0 if $X_1 + X_2 > c$

   (a) Find the value of c so that ϕ_2 has the same size as ϕ_1. (10%)

   (b) Is ϕ_2 a more powerful test than ϕ_1 ? Justify your answer. (8%)