國立中央大學八十五學年度碩士班研究生入學試題卷

所別：統計研究所　乙組　　　　科目：乙基礎數學　　　　　　　共 _1_ 頁　第 _1_ 頁

Note: No work No points.

1. Find the derivative of the following function at $\lambda = 0$: (20%)

$$f(\lambda) = \begin{cases} (y^\lambda - 1)/(\lambda x^{\lambda-1}) & \text{if } \lambda \neq 0 \\ x \ln y & \text{if } \lambda = 0 \end{cases}$$

2. (a) A positive definite matrix A has eigenvalues $\lambda_1 \geq \lambda_2 > \cdots \geq \lambda_n > 0$. Find Trace (A^{-1}). (10%)

   (b) X is an $n \times p$ matrix of rank $p(n > p)$. Find Trace $(X(X^TX)^{-1}X^T)$. (10%)

3. Maximize $x + y + 2z$ on the sphere $x^2 + y^2 + z^2 = 9$. (15%)

4. Find $\int_0^\infty a^{11} x^{10} e^{-ax} dx$. (15%)

5. Find the Taylor's Expansion at zero for $f(x) = 1/(1 - x^2), |x| < 1$ (15%)

6. I is the $n \times n$ identity and x is an $n \times 1$ vector with $\|x\| = 1$. Prove that $I - xx^T$ is a projection and find its projection space. (15%)