

系所別:

統計研究所

科目:

數理統計

[30%] 1. True or False (5 points each).

- a) Let X_1, X_2, \dots, X_n be independent and identically distributed (i.i.d.) random variables with probability density function (pdf) $f(x|\theta)$, where $\theta > 0$ is an unknown parameter. Then there always exists at least one sufficient statistic.
- b) An event with probability zero will never occur.
- c) Randomly select two balls without replacement from a box containing one red balls and two green balls. Let X_i denote the number of red ball selected on the i^{th} draw, $i = 1, 2$. Then X_1 and X_2 are identically distributed.
- d) Let X_1, X_2, \dots, X_n be a random sample from a Chi-square distribution of 50 degrees of freedom. Then $P(\bar{X} \leq 50)$ is approximately equal to 0.5, where \bar{X} is the sample mean.
- e) Let X_1, X_2, \dots, X_n be a random sample from a T distribution of 30 degrees of freedom, then \bar{X} and S^2 , the usual sample mean and sample variance, are independent.
- f) A continuous random variable has unique and continuous cumulative distribution function (cdf) and a discrete random variable also has unique cdf but discontinuous.

[10%] 2. Let $Y = \sum_{i=1}^n X_i$, where X_1, X_2, \dots, X_n are a random sample from a Poisson distribution with mean μ . Show that the variance of $\sqrt{Y/n}$ is essentially free of μ for large n .

[10%] 3. Let X be a random variable of a mixture of pdf's f_1 and f_2 with probability p_1 and p_2 , respectively, with conditional pdf,

$$f(x|\theta) = p_1 f_1(x|\theta) + p_2 f_2(x|\theta), \text{ given } \theta$$

where $0 < p_1, p_2 < 1$ and $p_1 + p_2 = 1$. Let $\pi(\theta)$ denote the pdf of θ . Show that the conditional distribution of θ given x is also of a mixture distribution.

[15%] 4. Let $Y_i \sim N(\alpha + x_i \beta, \sigma^2), i = 1, \dots, n$ be independent random variables, where x_i are given constants and α, β and $\sigma^2 > 0$ are unknown parameters. Find the maximum likelihood estimators (mle) of α, β , and σ^2 , if it is known that $0 < \alpha < 10$.

[10%] 5. An industrial concern runs two large plants. If the number of accidents (assumed to be of Poisson distribution with mean λ_1) during the last 5 weeks at plant 1 were 16, 18, 9, 22, 17 while the number of accidents (also assumed to be Poisson but with mean λ_2) during the last 6 weeks at plant 2 were 22, 18, 26, 30, 25, 28. Give a test and identify the p-value for $H_0 : \lambda_2 = 3/4 \lambda_1$ against $H_1 : \lambda_2 \neq 3/4 \lambda_1$.

[10%] 6. An industrial safety program was recently instituted in the computer chip industry. The average weekly loss (averaged over 1 month) in man-hours due to accidents (assumed to be normally distributed) in 10 similar plants both before and after the program are as follows:

Plant	1	2	3	4	5	6	7	8	9	10
Before	30.5	18.5	24.5	32	16	15	23.5	25.5	28	18
After	23	21	22	28.5	14.5	15.5	24.5	21	23.5	16.5

Determine at the α level of significance, whether the safety program has been proven to be effective. (Identify the distribution you use for the test statistic.)

[15%] 7. Let X_1, X_2, \dots, X_n be a random sample from a distribution with pdf

$$f(x|\theta) = \theta^2 x e^{-\theta x}, 0 < x < \infty,$$

where $\theta > 0$. Find the unbiased minimum variance estimator of θ .

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