國立中央大學100學年度碩士班考試入學試題卷

所別：光電科學與工程學系碩士班 不分組(一般生)　科目：近代物理　共 2 頁 第 1 頁

本科考試可使用計算器，廠牌、功能不拘　＊請在試卷答案卷（卡）內作答

參考用

1. S, S' and S'' are three inertial frames of reference. According to the observations made in S, frame S' moves rightward with velocity $\mathbf{u}=(u,0,0)$, whereas frame S'' moves upward with velocity $\mathbf{v}=(0,v,0)$. In addition, the coordinate axes of $S'(x',y',z')$ and S'' (x'',y'',z'') are parallel to the axes of $S(x,y,z)$, and their origins are coincide at the time origin $t=t'=t''=0$. The Lorentz transformation between S and S' is given by

$$x'=\frac{x-ut}{\sqrt{1-\frac{u^2}{c^2}}},\quad y'=y,\quad z'=z,\quad t'=\frac{t-\frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}}$$

(a) (7 pts) Find the Lorentz transformation between S' and S''.

(b) (7 pts) A rod of proper length l_0 rests on the x' axis of frame S'. Find its observed length in frame S''.

2. A particle of mass m is confined in a 2D box of size $a\times b$, defined by the potential energy:

$$V(x,y)=\begin{cases}0, & \text{if } 0<x<a \text{ and } 0<y<b\\ \infty, & \text{otherwise}\end{cases}$$

The time-dependent Schrödinger equation for this 2D system is given by

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+V(x,y)\psi$$

where $\psi=\psi(x,y,t)$ is the time-dependent wave function.

(a) (7 pts) Derive the time-independent Schrodinger equation using the method of "separation of variables" and find its solutions.

(b) (7 pts) Describe how to derive $\psi(x,y,t)$ if the initial wave function $\psi(x,y,0)$ is given.

3. Explain the following terms:

(3 pts) (a) Tunneling effect, (3 pts) (b) Meissner effect, (3 pts) (c) Equivalence principle, (3 pts) (d) Correspondence principle.

4. (10 pts) Calculate the uncertainty of particle position

$$\Delta x=\sqrt{\langle x^2\rangle-\langle x\rangle^2}$$

for the ground state wave function of harmonic oscillator

$$u_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{m\omega(x-x_0)^2}{2\hbar}\right)$$

Here x_0 is the location of the potential minimum of the oscillator system, and $\langle x\rangle$ denotes the expectation

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value of the position x.

5. (a) (5 pts) Please show and explain one physical phenomenon/process that can be observed in the universe and can evidence Einstein's famous formula of "$E=mc^2$", where E is the energy, m is the rest mass, and $c\sim3\times10^8$ m/s.

(b) (4 pts) Prove that for a massless particle its speed is always equal to c.

6. (a) (6 pts) How to generate, deflect, and focus an X-ray beam and how to measure the spectrum of the beam? Please design the experimental setups to illustrate your answers.

(b) (6 pts) How to generate, deflect, and focus an electron beam and how to measure the energy spectrum of the beam? Please design the experimental setups to illustrate your answers.

(c) (5 pts) Design an experiment to verify the wave nature of electrons (with kinetic energy of, say, 100 eV if you need the information).

參考用

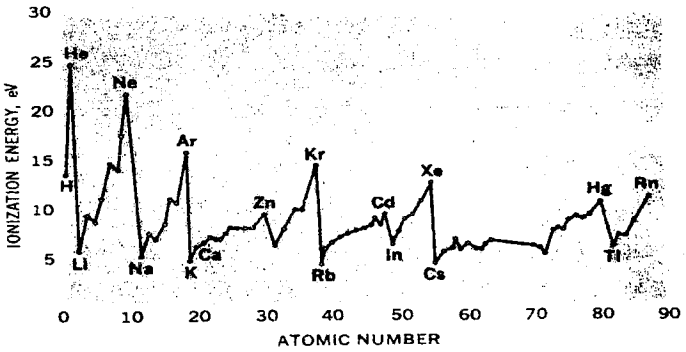

Fig. A Variation of ionization energy with atomic number

7. Figure A shows the ionization energies of the elements varied with the atomic number.

(a) (4 pts) Please write down the electron configurations of the elements in Period 2 of the periodic table (Hint: these elements are ${}_3$Li, ${}_4$Be, ${}_5$B, ${}_6$C, ${}_7$N, ${}_8$O, ${}_9$F, ${}_{10}$Ne).

(b) (5 pts) Take the Period 2 elements as an example, please explain why the ionization energy increases mainly with the increase of the atomic number of elements in the same period?

(c) (5 pts) Please explain why the ionization energy generally decreases as elements going down a group in the periodic table?

(d) (5 pts) According to Fig. A, please predict the trends in atomic size for elements in a period (say, Period 2) in the periodic table (leaving the noble gases out).

8. (5 pts) Expound the effects of the absorption, spontaneous emission, and stimulated emission processes in energy-level transitions on the laser generation.

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