國立中央大學 110 學年度碩士班考試入學試題

所別： 光電類　　共 2 頁　第 1 頁

科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答

Part A.

Choose the correct answer (50 %)（每題 5 分）

(1) For an equation given as $\frac{dy}{dx}=\frac{x^3y-y}{y^4-y^2+1}$ with $y(0)=1$, what is the relationship between x and y?

(A) x^4-4x-1 = y^4-$2y^2$+3log|y| (B) x^4-4x-1 = y^3-$2y^2$+4log|y| (C) x^4-2x-1 = y^4-$2y^2$+4log|y|
(D) x^4-4x-2 = y^4-$2y^2$+4log|y| (E) x^3-4x-1 = y^4-$2y^2$+4log|y| (F) x^4-4x-1 = y^4-$2y^2$+4log|y|
(G) x^4-4x-1 = y^4-$2y^3$+4log|y|.

(2) If $\frac{dy}{dx}=2y$, with y(0) = 1, what is y(x)? (A) e^x (B) $e^{0.5x}$ (C) e^{2x} (D) $2e^x$ (E) e^{4x} (F) e^{5x} (G) 1

(3) If $\frac{dy}{dx}=\frac{x}{y}$ and $y=10$ when $x=5$, what is y when $x=\sqrt{6}$? (A) 5 (B) 9 (C) $\sqrt{10}$
(D) $\sqrt{12}$ (E) 8 (F) 7 (G) 6

(4) If $y\frac{dy}{dx}=\frac{y^2+x^2}{x}$ and $y=1$ when $x=2$, what is y when $x=1$? (A)1.14 (B) -1.14 (C) 2.14
(D) -2.24 (E) 3.14 (F) -3.14 (G) None of the above.

(5) For the differential equation: $(x^2+4y)(dy/dx)+(2xy+1)=0$, what is the function $P(x,y)$ that satisfies the relationship: $\frac{\partial}{\partial x}\left[\frac{\partial P(x,y)}{\partial y}\right]=\frac{\partial}{\partial y}\left[\frac{\partial P(x,y)}{\partial x}\right]$? (A) $P(x,y)=x^2y+x+2y^2$ (B) $P(x,y)=x^2+x+2y^2$
(C) $P(x,y)=x^2y+x+y^2$ (D) $P(x,y)=xy+x+2y^2$ (E) $P(x,y)=2x^2y+x+2y^2$ (F) $P(x,y)=xy^2+x+2y^2$
(G) $P(x,y)=x^2y+2y^2$

(6) If $P(f,t)$ is the general solution of the function: $(ft-t^2)\frac{df}{dt}=f^2$, what is $P(2,1)$?
(A) 2.31 (B) 3.31 (C) 0 (D) 1.31 (E) 4.31 (F) 5.31 (G) 6.31

(7) If $\frac{dy}{dx}=\frac{2+\sin(x)}{3(y-1)^2}$ with $y(0)=2$, what is $y(\pi)$? (A) 5.1 (B) 1.1 (C) 2.1 (D) 6.1 (E) 7.1
(F) 3.1 (G) 4.1

(8) If $\frac{dy}{dt}=2y$, with $y(0)=-1$, what is $y(0.5)$? (A) 0.368 (B) 2.718 (C) 5.436 (D) 0.736 (E) 8.155
(F) 1.104 (G) 1.472

(9) The amount, A(t), of carbon 14 in an archaeological sample decays exponentially with time, and A(t) can be estimated by the equation: $A(t)=Ce^{-kt}$, where C and k are constants. If the half-life of carbon 14 is 5570 years, what is k? (A) 2.21×10^{-3} year^{-1} (B) 8.54×10^{-4} year^{-1} (C) 1.23×10^{-5} year^{-1}
(D) 7.24×10^{-3} year^{-1} (E) 1.28×10^{-3} year^{-1} (F) 9.24×10^{-5} year^{-1} (G) 1.24×10^{-4} year^{-1}

(10) If $\frac{dy}{dx}=-4x+2y$ and $y=1$ when $x=0$, what is x when $y=3$? (A) -3 (B) 2 (C) 1
(D) 0 (E) -2 (F) 3 (G) None of the above

注意:背面有試題

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Part B.

Solving the following problems (50%) (每一大題 10 分，請務必寫出計算過程)

(11) (10pt) Find the rank of A: $A=\begin{pmatrix} 3 & 1 & -3 & 1 \\ 2 & 3 & 0 & 1 \\ -2 & -4 & 2 & 0 \\ 0 & 2 & 1 & 0 \end{pmatrix}$.

(12) (10pt) Calculate the matrix function $f(M)=M^{30}+3M^{19}$, where the matrix M is defined as

$$M=\begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}$$

(13) (10pt)For a quadratic form $Q=x^TAx$, if for any nonzero $x=[x_1,x_2,\cdots,x_n]^T$ we always have $Q>0$, then this quadratic form is positive definite.

For $n=3$ and $A=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 5 & -2 & 4 \end{pmatrix}$, is the quadratic form $Q=x^TAx$ positive definite?

(14) (10pt) A surface is parameterized by u and v as

$$x=2\cosh v\cos u$$
$$y=3\cosh v\sin u$$
$$z=5\sinh v$$

Here $0\le u\le 2\pi$, $-\infty<v<\infty$.

(a) (5pt) Express the surface equation in the form $f(x,y,z)=1$. Find $f(x,y,z)$.

(b) (5pt) Find the unit normal vector $\hat{n}$ of the surface at an arbitrary point.

(15) (10pt) Find the Fourier transform of the function: $f(x)=xe^{-|x|}$.

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