

所別： 光電類

第 1 頁 / 共 2 頁

科目： 工程數學

\*本科考試可使用計算器，廠牌、功能不拘

Solve the following problems : 計算題(無計算過程者不予計分)

1. (a) (5%) Are  $\sin(x)$  and  $e^x$  linearly independent? Justify.  
(b) (5%) Are  $e^x$  and  $e^{x+2}$  linearly independent? Justify.
2. (10%) Find the general solution of  $y'' + 5y' + 6y = 2x + 1$ .
3. (10%) Find the solution of  $xy'' + y' = 0$ .
4. (10%) Consider a mass-spring system:  
 $x''(t) + x(t) = f(t)$ , IVs :  $x(0) = 0, x'(0) = 0$ , where  $f(t) = 1$ , if  $1 \leq t < 5$   
and zero otherwise. Find the solution of the system.
5. (10%) Let  $f(t) = |t|$ , for  $-1 < t \leq 1$ , extended periodically. Compute the  
Fourier series of  $f(t)$ .

注意：背面有試題

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6. (10%) Consider the vector function  $\vec{F}(x, y, z) = [x^2y, -xy^2, xyz]$  and the surface  $S$  being the hemisphere  $x^2 + y^2 + z^2 = 4, z \geq 0$ . Evaluate  $\iint_S (\nabla \times \vec{F}) \cdot \vec{n} dA$ .

7. (10%) Solve the linear system 
$$\begin{bmatrix} -1 & 1 & 1 \\ 2 & -6 & 3 \\ 2 & 6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ 8 \end{bmatrix}.$$

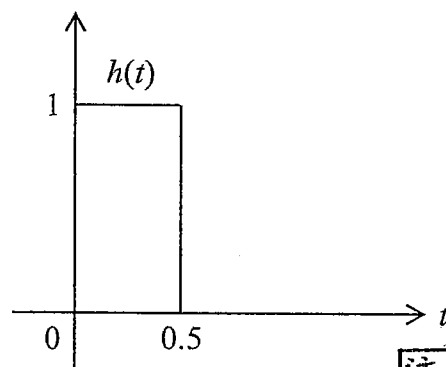
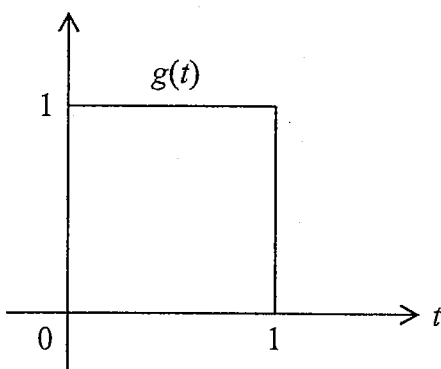
8. (10%) Given the matrix  $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ , compute  $A^{17}$ .

9. Denote the Fourier transform of  $f(t)$  as  $F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$ .

- (a) (5%) Determine the Fourier transform of  $f(t) = \cos^2(2\pi t)$ .  
 (b) (5%) Find the Fourier transforms of  $g(t)$  and  $h(t)$  in the figures.  
 (c) (10%) The convolution of  $g(t)$  and  $h(t)$  are defined as

$$(g * h)(t) = \int_{-\infty}^{\infty} g(\tau)h(t - \tau)d\tau$$

Sketch  $(g * h)(t)$  and find the Fourier transform of  $(g * h)(t)$ .



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