國立中央大學114學年度碩士班考試入學試題

系所： 光電類

科目： 工程數學

＊本科考試可使用計算器，廠牌、功能不拘

Solve the following problems： 計算題（無計算過程者不予計分）

1. (10%) Considering a rectangular aperture illuminated by a normally incident plane wave with unit amplitude, we thus have an amplitude transmittance given by:

$$U(\xi,\eta)=\begin{cases}1, & \text{if } \left(-\frac{a}{2}\le\xi\le\frac{a}{2}\right)\cap\left(-\frac{b}{2}\le\eta\le\frac{b}{2}\right);\\ 0, & \text{otherwise.}\end{cases}$$

Find out the Far-field diffraction pattern, $I(x,y)$, by using the Fraunhofer diffraction equation:

$$U(x,y)=\frac{e^{jkz}e^{j\frac{k}{2}(x^2+y^2)}}{j\lambda z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(\xi,\eta)\exp\left[-j\frac{2\pi}{\lambda z}(x\xi+y\eta)\right]d\xi d\eta,$$

where $I(x,y)=|U(x,y)|^2$.

2. (5%) The particular solution of $y''-3y'+2y=0$ with initial values $y(0)=1$ and $y'(0)=0$ is given by: (a) $e^{2t}(2-e^{-t})$; (b) $-e^{2t}(1-2e^{-t})$; (c) $\frac{1}{2}e^{3t}(3-e^{-2t})$; (d) $-\frac{1}{2}e^{3t}(1-3e^{-2t})$; (e) None of the above.

3. (5%) The particular solution of $y''-2y'+2y=0$ with initial values $y(0)=1$ and $y'(0)=0$ is given by: (a) $e^{t}(\cos t+\sin t)$; (b) $e^{t}(\cos t-\sin t)$; (c) $e^{t}(\cos 2t-\frac{1}{2}\sin 2t)$; (d) $e^{2t}(\cos t-2\sin 2t)$; (e) None of the above.

4. (6%) Find the general solution of the nonhomogeneous ordinary differential equation: $y''+5y'+6y=2e^{-t}$.

5. (2%) The particular solution of $y''+5y'+6y=2e^{-t}$ with initial values $y(0)=0$ and $y'(0)=0$ is given by: (a) $y(t)=e^{-t}(1-2e^{-t}+e^{-2t})$; (b) $y(t)=e^{t}(1-4e^{-3t}+3e^{-4t})$; (c) $y(t)=e^{3t}(3-4e^{-t}+e^{-4t})$; (d) $y(t)=e^{3t}(1-2e^{-t}+e^{-2t})$; (e) None of the above.

6. (2%) The particular solution of $y''+5y'+6y=2e^{-t}$ with initial values $y(0)=1$ and $y'(0)=0$ is given by: (a) $y(t)=e^{-t}(1+e^{-t}-e^{-2t})$; (b) $y(t)=e^{t}(1-e^{-3t}+e^{-4t})$; (c) $y(t)=e^{3t}(1-e^{-t}+e^{-4t})$; (d) $y(t)=-e^{3t}(1-e^{-t}-e^{-2t})$; (e) None of the above.

注意:背面有試題

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7. (10%) Consider an inhomogeneous constant-coefficient second-order ordinary differential equation with initial conditions given by $ay''+by'+cy=g(t)$, $y(0)=y_0$, $y'(0)=u_0$. Determine the solution for Y = Y(s) in terms of Laplace transform of $g(t)$.

8. (10%) Find two independent power series solutions of $y''-y=0$. Show that the corresponding general solution is given by $y(x)=a_0\cosh x+a_1\sinh x$.

9. (10%) Given four points in 3D space: (2, 6, 1), (3, 1, 7), (4, 3, 9), and (5, 2, 8) as the vertices of a tetrahedron (四面體), find the volume.

10. (10%) Given the vector function $\vec{F}(x,y,z)=[2yz-x^3, 2xz-y^3, 2xy-z^3]$ and the surface S of $x^2+y^2+z^2\le 9$, $z\ge 0$, evaluate $\iint_S \vec{F}\cdot\vec{n}dA$.

11. Consider the linear transformation $\vec{y}=\mathbf{A}\vec{x}$, where $\vec{x}=[x_1 \quad x_2]^T$ and $\vec{y}=[y_1 \quad y_2]^T$ are Cartesian coordinates in R^2.

(a) (3%) Find the matrix $\mathbf{A}_1$ for stretching along the x_2-axis by a factor of 3.

(b) (3%) Find the matrix $\mathbf{A}_2$ for counter clockwise rotation by the angle $\pi/6$ about the origin.

(c) (2%) Find the matrix $\mathbf{A}_3$ for a point undergoes the processes in (a) and then (b).

(d) (2%) A unit circle $x_1^2+x_2^2=1$ undergoes the processes in (a) and then (b). Draw the resulting figure.

12. Consider the matrix $\mathbf{A}=\begin{bmatrix}5 & 4 & -4\\ -1 & -1 & 3\\ 7 & 6 & -4\end{bmatrix}$.

(a) (2%) Find the determinant of **A**.

(b) (8%) Find the eigenvalues and the corresponding eigenvectors of **A**.

(c) (4%) Find a matrix **X** that diagonalizes **A** to a diagonal matrix **D**. (需寫出 **D**)

(d) (4%) Find $\mathbf{X}^{-1}$, the inverse of **X**.

(e) (2%) Express $\mathbf{A}^8$ in terms of **X** and **D**. (不需要算出數值)

注意:背面有試題