

國立中央大學98學年度碩士班考試入學試題卷

所別：光電科學與工程學系碩士班 一般生 科目：工程數學 共 1 頁 第 1 頁

*請在試卷答案卷(卡)內作答

1. (5%) Derive the Taylor's expansion of $\ln(1+x)$ in increasing powers of x .
2. (5%) Evaluate the value of $\ln(1.1)$ with the inaccuracy less than 10^{-3} .
3. (10%) An $n \times n$ square matrix \mathbf{A} is diagonalizable if there exists an invertible matrix \mathbf{M} such that $\mathbf{M}^{-1}\mathbf{A}\mathbf{M}$ is a diagonal matrix. Now, $\lambda_1, \lambda_2, \dots, \lambda_n$, are the eigenvalues of the matrix \mathbf{A} . Find out the k powers of the matrix \mathbf{A} (i.e. $\mathbf{A}^k = ?$) in terms of $\lambda_1, \lambda_2, \dots, \lambda_n$ and \mathbf{M} .
4. (10%) When $A = \begin{pmatrix} 0 & 0.1 \\ 0.1 & 0 \end{pmatrix}$, calculate $\ln(\mathbf{I} + \mathbf{A})$. Note: \ln is defined by the Taylor's expansion of the logarithm.
5. There is an ellipse $(x/a)^2 + (y/b)^2 = 1$.
 - (a) (10%) Find the area of the ellipse;
 - (b) (10%) Find the inscribed rectangle of maximum area.
6. (15%) Find the general solution of the ODE: $y''' + 2y'' - 5y' - 6y = -12\cos^2 x$.
7. (10%) Solve the initial value problem by using the Laplace transform:

$$y'' + 9y = 3\delta(t - \pi), \quad y(0) = y'(0) = 1.$$
8. Consider $f(x) = \begin{cases} 1 - |x|, & \text{for } |x| < 1; \\ 0, & \text{otherwise.} \end{cases}$
 - (a) (10%) Determine the Fourier transform of $f(x)$, $\mathcal{F}\{f(x)\}$;
 - (b) (5%) Sketch $f(x)$ and $\mathcal{F}\{f(x)\}$.
9. Given Gamma function, $\Gamma(\nu) = \int_0^\infty e^{-t} t^{\nu-1} dt$ ($\nu > 0$), and Bessel function of the first kind of order ν , $J_\nu(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m! \Gamma(\nu + m + 1)} \left(\frac{x}{2}\right)^{2m+\nu}$, show that

(a) (5%) $\Gamma(1/2) = \sqrt{\pi}$;

(b) (5%) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

參考用