國立中央大學99學年度碩士班考試入學試題卷

所別：光電科學與工程學系碩士班 不分組(一般生)　科目：近代物理　共 2 頁 第 1 頁
不分組(在職生)

＊請在試卷答案卷（卡）內作答

＊本科考試禁用計算器

1. (5 pts) Please sketch (a) classical and (b) relativistic **total energy**-velocity plots of a particle of mass m in a free space (no gravity) in the same figure. Please carefully label the axes, points that the curves intersect with the axes and as the velocity equal c.

2. (5 pts) Figure A is an 1D potential well as a function of position. The potential well is symmetric to the origin. If E4 and E7 are two eigenstate energies which corresponding to quantum number 4 and 7. Please **sketch** the corresponding wave functions. Please carefully label the axes and the position a and b in your sketch. Also, please make a brief explain the wave function behavior in the region $a<x<b$.

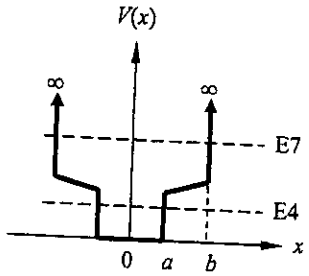

Fig. A. 1D potential well for problem 2.

3. If another potential wall is only the positive half of the Fig. A. Please answer the following questions.
(a) (5 pts) How many eigenstates in this new potential well has energy equal or lower than E7 in problem 2?
(b) (5 pts) Explain how you get the number in 3(a). **(limited to 50 words or less)**

4. (5 pts) Please list all possible radiative allowed downward transitions (go to lower energy states) for an electron at 4p orbital of an He^+ ion. (Please use the symbol like $6s \rightarrow 5p$.)

5. A potential energy $V(x,y)$ is as shown in Fig. B. A particle with mass m is placed in this square box potential. We can use n_x and n_y as the quantum numbers of the wavefunction along x- and y-axis. We can use 2D time-independent Schrodinger equation and separation of variable to solve the steady state wavefunctions of this system as well as the corresponding energy levels.
(a) (5 pts) Based on uncertainty principle, the lowest possible kinetic energy of this particle is_____.
(Fill-in-the-blank problems, please just give the result)
(b) (5 pts) By solving time-independent Schrodinger equation, the general form of the steady state wavefunction is _____. (Fill-in-the-blank problems, please just give the result)
(c) (5 pts) Base on the result from (b), we can obtain the general form of the energy of the corresponding wavefunction as _____. (Fill-in-the-blank problems, please just give the result)
(d) (5 pts) Please give another wavefunctions which has the same kinetic energy as $(n_x,n_y)=(5, 5)$.

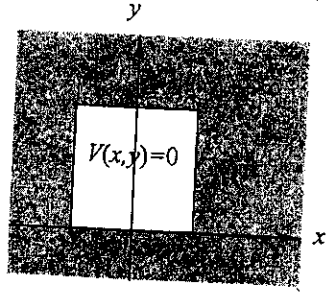

Fig. B. Potential energy distribution in Problem 7.

參考用

注意：背面有試題

6. (5 pts) All the wavefunction we obtained from the time-independent Schrödinger equation are steady states or the solution which will not change with time. Why do we still observe spontaneous emission? **(limited to 100 words or less)**

7. (8 pts) Please suggest two measurement methods to acquire the atomic spectra (energy levels) of a gas (say, helium) using electron beams.

8. Questions on uncertainty principle:

(a) (7 pts) Use of the concept of the Heisenberg's microscope to prove the uncertainty principle regarding the relationship between the position and momentum of a particle.

(b) (5 pts) Use the uncertainty relation you obtained in (a) to estimate the minimum kinetic energy of an electron that can confine in a hydrogen atom (radius~ 0.5 Å). Please comment on the consistency between the result you just calculated and the known kinetic energy of an electron in a ground-state hydrogen atom (~13.6 eV). (Planck's constant h~6.6×10^{-34} J-s, electron mass~ 9×10^{-31} kg)

(c) (4 pts) Derive the Heisenberg uncertainty principle for the relationship between the time and energy of a photon directly from the position-momentum uncertainty relation you have obtained in (a).

(d) (4 pts) Use of the Heisenberg uncertainty principle to evidence that there exists a zero-point energy in the universe.

9. The resolving power of a microscope is limited by the diffraction of the wave source used in the system.

(a) (3 pts) Please illustrate why an electron microscope has a much better resolution than a typical optical microscope?

(b) (8 pts) Design experimental setups to deduce the wavelength of an electron source having a kinetic energy of, say, 100 keV *without* using the known de Broglie's formula. Sketch and illustrate your design schemes. (Hint: you can mainly use a X-ray tube, an electron gun, a Ni crystal, and any other scientific instruments in your designs)

(c) (5 pts) Sketch the structure scheme of an electron microscope to illustrate its working principle.

10. (6 pts) Find the criterion for a star of mass M and radius R to be a black hole in the nonrelativistic limit.