

國立中央大學99學年度碩士班考試入學試題卷

所別：光電科學與工程學系碩士班 不分組(一般生)
不分組(在職生)

科目：工程數學 共 / 頁 第 / 頁

*請在試卷答案卷(卡)內作答

*本科考試禁用計算器

(10%) 1. Find the general solution of the ODE: $x^3 y''' - x^2 y'' - 2xy' + 6y = 7x^{-2}$.

(15%) 2. Solve the following system, where $\text{step}(t)$ is the Heaviside step function.

$$\begin{cases} 2\frac{dx}{dt} + x - y = \text{step}(t-3) \\ 3\frac{dy}{dt} - x + 2y = 0 \end{cases} \quad \text{and } x(0) = y(0) = 0.$$

(10%) 3. Determine the Fourier transform of $f(t) = \sin(3t) \exp(-5|t|)$.

(15%) 4. Solve the initial value problem: $y'' + xy' - 2y = 1$, $y(0) = 1$, $y'(0) = 2$.

5. For the inverse trigonometric function $\sin^{-1} x$

(10%) (a) Expand $\sin^{-1} x$ in a Taylor series at $x = 0$.

(5%) (b) Estimate the value of $\sin^{-1} 0.1$ with the inaccuracy less than 10^{-4} .

(10%) 6. Given $K = \begin{pmatrix} 0 & 0 & i \\ -i & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$, find the proper choice of n such that $K^n = I$.

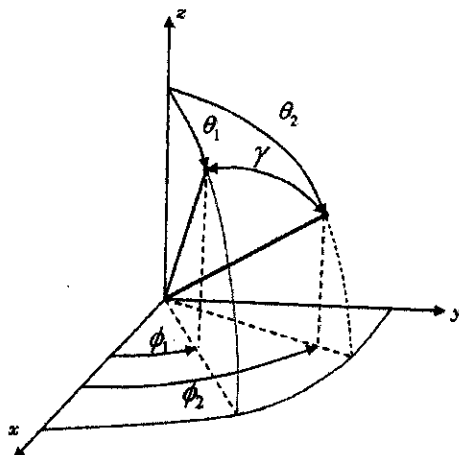
7. A function $F(z)$ is defined as $F(z) \equiv \int_0^{\infty} e^{-t} t^{z-1} dt$ for all z with positive real part.

(5%) (a). Derive the relation between $F(z+1)$ and $F(z)$

(5%) (b). Show that the definition can be rewritten as $F(z) \equiv 2 \int_0^{\infty} e^{-t^2} t^{2z-1} dt$, $\Re(z) > 0$

(5%) (c). Find the value of $F(\frac{1}{2})$.

(10%) 8. The direction of one vector is given by the angles θ_1 and ϕ_1 . For a second vector the corresponding angles are θ_2 and ϕ_2 . Show that the cosine of the included angle γ is given by $\cos \gamma = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\phi_1 - \phi_2)$



參考用