所別：天文研究所碩士班 不分組(一般生)　科目：應用數學　共 2 頁 第 1 頁
天文研究所碩士班 不分組(在職生)

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

參考用

1 Angle between two vectors

Two vectors, $\vec{a}$ and $\vec{b}$, are given as

$$\vec{a} = (a_x, a_y, a_z),$$
$$\vec{b} = (b_x, b_y, b_z).$$

Find the angle between these two vectors. (5 points)

2 Matrix

Consider a matrix A,

$$A = \begin{pmatrix} 1 & 6 \\ 2 & -3 \end{pmatrix}.$$

(a) Calculate the determinant of the matrix A. (5 points)

(b) Find the inverse matrix of A. (5 points)

(c) Calculate the eigenvalues of the matrix A. (5 points)

(d) Find the eigenvectors of the matrix A. (5 points)

(e) Find the matrix P to diagonalize the matrix A, and calculate $P^{-1}AP$. (5 points)

(f) Calculate A^n. (5 points)
(Hint: Try to calculate $(P^{-1}AP)^n$.)

3 Taylor series expansion

Find Taylor series expansion about a point $x = 0$ for following functions.

(a) e^x (5 points)

(b) $\dfrac{1}{1-x}$ $(|x| < 1)$ (5 points)

4 Integrals

Perform following integrals.

(a) $\displaystyle\int \frac{e^{-x}}{1+e^{-x}} dx$ (5 points)

(b) $\displaystyle\int x \sin x dx$ (5 points)

(c) $\displaystyle\int_1^2 \frac{\ln x}{x} dx$ (5 points)

(d) $\displaystyle\int_{-\infty}^{\infty} e^{-x^2} dx$ (5 points)
(Hint: You may use polar coordinate for your calculation.)

注意:背面有試題

5 Differential equations

Find the general solutions for following differential equations. Note that k is a constant.

(a) $\frac{dy}{dx} = -kx$ (10 points)

(b) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12 = 0$ (10 points)

6 Wave equation

Suppose you have following four equations,

$$\begin{aligned} \mathrm{div}\vec{E} &= \vec{\nabla}\cdot\vec{E} = 0, \\ \mathrm{div}\vec{H} &= \vec{\nabla}\cdot\vec{H} = 0, \\ \mathrm{rot}\vec{E} &= \vec{\nabla}\times\vec{E} = -\mu_0\frac{\partial\vec{H}}{\partial t}, \\ \mathrm{rot}\vec{H} &= \vec{\nabla}\times\vec{H} = \epsilon_0\frac{\partial\vec{E}}{\partial t}. \end{aligned}$$

Here, $\vec{\nabla}$ is a vector differential operator. Starting from above four equations, derive the wave equation of following form,

$$\left(\vec{\nabla}^2 - \frac{1}{\sqrt{1/(\epsilon_0\mu_0)}^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \vec{0}.$$

Hint: You may use following formula if needed.

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \left(\vec{A}\cdot\vec{C}\right)\vec{B} - \left(\vec{A}\cdot\vec{B}\right)\vec{C}$$

(15 points)