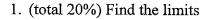
國立中央大學104學年度碩士班考試入學試題

天文研究所碩士班 不分組(在職生)

本科考試禁用計算器

*請在答案卷 (卡) 內作答



(i) (4%)
$$\lim_{x \to \infty} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$$
 (ii) (4%) $\lim_{x \to 2} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$

(iii) (4%)
$$\lim_{x\to 0} \frac{\cosh x - 1}{1 - \cos 2x}$$

- (iv) (4%) $\lim_{x\to\infty} x^n e^{-x}$, where n is a finite positive integer
- (v) (4%) $\lim_{x\to 0^+} x^x$
- 2. (total 15%) If a curve is represented by a continuously differentiable function

$$\vec{r} = \vec{r}(t)$$

- (i) (5%) Show that the length of the curve l between t = a and t = b where b > a is $l = \int_a^b \sqrt{\vec{r} \cdot \vec{r}} dt$ where $\dot{\vec{r}} = \frac{d\vec{r}}{dt}$
- (ii) (5%) If the curve is on the x-y plane and y can be written as a function of x, using the equation in (i), show that the length of the curve between $x = x_1$ and $x = x_2$ where $x_2 > x_1$ can be written as

$$l = \int_{x_1}^{x_2} \sqrt{1 + (y')^2} dx$$
, where $y' = \frac{dy}{dx}$

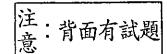
(iii) (5%) If the curve is represented in polar coordinate (ρ, θ) and the radial variable ρ can be written as a function of angular variable θ , that is $\rho = \rho(\theta)$, using the equation in (i), show that the length of the curve between $\theta = \theta_1$ and $\theta = \theta_2$ where $\theta_2 > \theta_1$ can be written as

$$l = \int_{\theta_{l}}^{\theta_{2}} \sqrt{\left(\rho\right)^{2} + \left(\frac{d\rho}{d\theta}\right)^{2}} d\theta$$

3. (total 15%) The Gaussian distribution is described as

$$P(x) = A \cdot \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$
 where A is the normalization constant and $-\infty < x < \infty$

(i) (5%) Find the normalization constant A so that $\int_{-\infty}^{\infty} P(x) dx = 1$



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所別:天文研究所碩士班 不分組(一般生) 科目:應用數學 共 2 頁 第 2 頁 天文研究所碩士班 不分組(在職生)

本科考試禁用計算器

*請在答案卷(卡)內作答

The expectation value of a function f(x) is defined as

$$\langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)P(x)dx$$

- (ii) (5%) calculate the mean: $\langle x \rangle$ and
- (iii) (5%) variance: $\langle (x \langle x \rangle)^2 \rangle$.



- 4. (total 15%) For an $n \times n$ matrix \mathbf{A} whose matrix elements are $\left[\mathbf{A}\right]_{ij} = a_{ij}$, its adjoint matrix is defined as \mathbf{A}^{\dagger} whose matrix elements are $\left[\mathbf{A}^{\dagger}\right]_{ij} = a_{ji}^{*}$ where a_{ji}^{*} is the complex conjugate of a_{ji}
 - (i) (5%) show that $(\mathbf{A}\mathbf{B})^{\dagger} = \mathbf{B}^{\dagger}\mathbf{A}^{\dagger}$
 - (ii) (5%) show that $trace(\mathbf{A}^{\dagger}\mathbf{A}) \ge 0$
 - (iii) (5%) A is Hermitian matrix if $A^{\dagger} = A$. If A and B are Hermitian matrices and AB BA = iC, show that C is a Hermitian matrix
- 5. (total 20%) A triangular wave is given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < \pi \\ -x & \text{for } -\pi < x < 0 \end{cases} \text{ and } f(x+2\pi) = f(x)$$

- (i) (10%) Represent f(x) by a Fourier series
- (ii) (10%) Use the result from (i) to calculate $\sum_{n=0}^{\infty} \frac{1}{(2n+1)}$
- 6. (15%) Show that

$$\sum_{n=0}^{N-1} \cos nx = \frac{\sin \frac{Nx}{2}}{\sin \frac{x}{2}} \cos \left[\frac{(N-1)x}{2} \right]$$

$$\sum_{n=0}^{N-1} \sin nx = \frac{\sin \frac{Nx}{2}}{\sin \frac{x}{2}} \sin \left[\frac{(N-1)x}{2} \right]$$

注:背面有試題