國立中央大學 110 學年度碩士班考試入學試題

所別： 天文研究所 碩士班 不分組(一般生)　　共 1 頁 第 1 頁

科目： 物理與天文

計算題及問答題

本科考試禁用計算器

＊請在答案卷(卡)內作答

※計算題需計算過程，無計算過程者不予計分

(1) (10 points)
The spectrum of a star can tell us some of its important properties. Choose three different types of properties that you can get from the spectrum of a star. Describe the physics behind each of your choices.

(2) (20 points)
(a) (6 points) Describe and explain the phenomena of interference and diffraction in the context of wave theory of light.
(b) (14 points) A thin film of thickness 500 nm is placed in vacuum. A visible light source with wavelengths from 360 nm to 720 nm is incident normally on the thin film and found that 400 nm and 600 nm are missing in the reflected light. Find the minimum possible refractive index of the thin film. For this refractive index, what wavelengths are reflected most strongly from the thin film? What wavelengths are transmitted most strongly through the thin film?

(3) (20 points)
The Sun and the Moon exert tidal force on the Earth.
(a) (10 points) Consider the Earth as a sphere of mass with radius $R_{\rm E}$. Derive, with the help of a diagram, the tidal force exerted by the Moon on the Earth's surface.
[Hint: assume the distance between the Moon and the Earth $d_{\rm EM}$ is much larger than $R_{\rm E}$.]
(b) (10 points) The ratio between the tidal force exerted by the Moon to that exerted by the Sun is 7 to 3. Estimate the ratio of the average density of the Moon to that of the Sun.
[State clear the assumptions you made and the extra information (if any) that you used.]

(4) (30 points)
(a) (10 points) In a polar coordinate system, the unit vector in the direction of r is $\hat{\mathbf{e}}_r$ and in the direction of ϕ is $\hat{\mathbf{e}}_\phi$. The coordinates of a particle in this coordinate is $(r(t), \phi(t))$. What is its position vector of the particle? Work out the velocity and acceleration of the particle in the directions $\hat{\mathbf{e}}_r$ and $\hat{\mathbf{e}}_\phi$.

The Sun is at the origin of a polar coordinate system. A planet is orbiting the Sun in elliptical orbit. Its distance from the Sun is r. The angle between its position vector and the semi-major axis of the orbit is ϕ.
The three Kepler's laws can be expressed as:
First law: $r = \dfrac{b^2}{a(1+\epsilon\cos\phi)}$; Second law: $\dfrac{r^2}{2}\dfrac{\mathrm{d}\phi}{\mathrm{d}t} = \dfrac{\pi ab}{P}$; Third law: $P^2 = a^3$;
where P is the orbital period, ϵ is the eccentricity, a is the semi-major axis, and $b = a\sqrt{1-\epsilon^2}$ is the semi-minor axis.
(b) (5 points) Show that the acceleration in the direction $\hat{\mathbf{e}}_\phi$ is zero.
(c) (15 points) Show that the acceleration in the direction $\hat{\mathbf{e}}_r$ is inversely proportional to r^2.

(5) (20 points)
(a) (6 points) Describe uncertainty principle in quantum physics. A particle is confined within a box which has a size of order d. Give an approximate expression for the kinetic energy of the particle (due to quantum effect) in non-relativistic regime.
(b) (7 points) A sphere of ionized nuclei and electrons has radius R and mass M (the sphere is neutral as a whole). Suppose the sphere is cold enough that the internal energy of the sphere is dominated by the quantum motion of electrons. If the total kinetic energy of the electrons is roughly the same as the total Coulomb binding energy, the relation between the radius and the mass can be approximated as $R \propto M^\alpha$. Find α.
(c) (7 points) Now if the total kinetic energy of the electrons in (b) is roughly the same as the total gravitational energy of the sphere (neglect Coulomb interaction), once again the relation between the radius and the mass can be approximated as $R \propto M^\beta$. Find β.