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參考用

Modern Physics

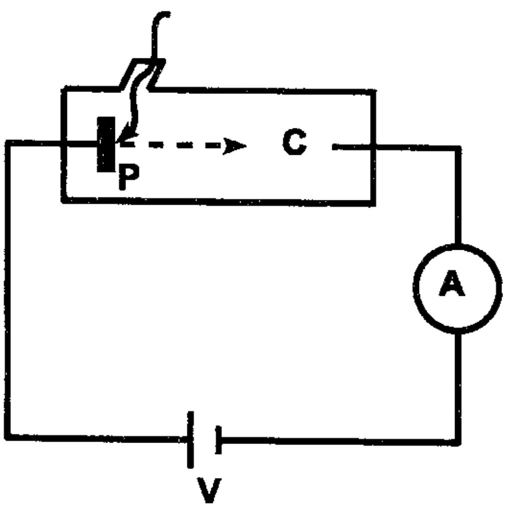
6 PROBLEMS. Show your calculation steps clearly

1. Light strikes a plate in an evacuated chamber. The result is that electrons are emitted from the plate P and are collected by C.

   (a) In the process a single photon gives up all its energy to a single electron. The maximum possible kinetic energy of the photoelectrons, $\frac{1}{2}mv_{max}^2$, is related to the energy of a photon, hf, by $hf = \frac{1}{2}mv_{max}^2 + \phi$. What is the physical meaning of ϕ? (5 points)

   (b) Plot the relation between the current measured by A and the applied voltage V at two different light intensity I_1 and I_2 ($I_2 > I_1$). Discuss your curves, especially discuss why the current vanishes at the same applied voltage V_0 for both curves. (5 points)

   (c) Discuss why an experimentalist can measure the Planck's constant h from the relation between V_0 and f. (5 points)



2. Suppose an electron with energy $E_1 = 8$ GeV (1 GeV = 10^9 eV) is smashed against a positron with energy $E_2 = 3.5$ GeV in a laboratory, and only a single particle is

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produced at rest in the center-of-mass frame in such an event. Note that the positron is the anti-particle of electron and thus has the same mass.

(a) What is the mass of the produced particle? (10 points)

(b) What is the speed of the produced particle in the laboratory frame? You may express your result in units of the speed of light, c. (5 points)

3. (a) Discuss why classical theory fails to predict the heat capacity of cavity radiation. (5 points)

(b) To solve this puzzle, Planck assumes that the energy of a monochromatic electromagnetic wave with a given polarization can only take discrete values $\epsilon = n\hbar\omega$, where n =0, 1, 2,..., $\hbar = h/2\pi$, ω is the angular frequency of electromagnetic wave. Given that in thermal equilibrium, the probability for this wave to have energy ϵ is proportional to $\exp(-\epsilon/k_BT)$, where k_B is Boltzmann constant. Calculate the thermal energy $\langle\epsilon\rangle$ associated with this wave at temperature T. (5 points)

(c) Show that your result in (b) reduces to k_BT when $\frac{\hbar\omega}{k_BT} \ll 1$. When $\frac{\hbar\omega}{k_BT} \gg 1$, the result becomes $\langle\epsilon\rangle \approx \hbar\omega\exp(-\frac{\hbar\omega}{k_BT})$. (5 points)

(d) Discuss how the result in (c) helps Planck to explain finiteness of the heat capacity of cavity radiation. (5 points)

4. Suppose a radio station uses a 50 kW broadcasting antenna to emit radio waves at a frequency of 100 MHz. Consider the ideal situation where the antenna can be approximated as a point source and there is no reflection from the ground. If a person is standing at a distance of 10 km away from the antenna.

(a) What is the momentum and energy of each photon? (5 points)

(b) Determine the flux of photons received by the person in the SI units. (5 points)

(c) Explain whether the quantum nature of the radiation is important for the stereo of the person. (5 points)

5. Here is a simple way to estimate the ground-state energy for a particle in a potential $U(x)$. From Heisenberg's uncertainty principle, the kinetic energy in the ground state

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is roughly $K = \frac{p^2}{2m} \sim \frac{\hbar^2}{2m\Delta x^2}$. The ground state energy of the particle is obtained by minimizing $K + U(\Delta x)$ with respect to Δx.

(a) Use this method to estimate the ground-state energy of a particle in a one-dimensional potential $U = \frac{1}{2}kx^2$. (5 points)

(b) Use this method to estimate the ground-state energy of a hydrogen atom, and give your result in eV. The permittivity constant $\epsilon_0 = 9 \times 10^{-12} C^2/N \cdot m^2$. (5 points)

(c) Estimate the radius of ground-state orbit for an electron in a hydrogen atom, and give your result in Å. (5 points)

6. Consider a one-dimensional simple harmonic oscillator of mass m and charge q. Suppose the system is placed in a static electric field of strength E. The Hamiltonian of this oscillator is given by

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 - qE\hat{x} .$$

The ground-state wave function when $E = 0$ is given by

$$\psi_0(x) = \frac{1}{(\sqrt{\pi}x_0)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x}{x_0}\right)^2\right] ,$$

where $x_0 = \sqrt{\hbar/(m\omega)}$.

(a) For a constant electric field E, find the energy levels for all states. (10 points)

(b) Determine the most likely position of the oscillator in the ground state and give a physical interpretation for your result. (10 points)

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TABLE I: The following physical constants and conversion factors may be useful in your numerical calculations.

Quantity	Symbol	Value
speed of light	c	3×10^{8} m/s
electron charge	e	1.6×10^{-19} C
reduced Planck constant	$\hbar$	10^{-34} J s
electron mass	m_e	9.1×10^{-31} kg $= 0.511$ MeV/c^2
proton mass	m_p	1.673×10^{-27} kg $= 938.27$ MeV/c^2
neutron mass	m_n	1.675×10^{-27} kg $= 939.56$ MeV/c^2
fine structure constant	α_{EM}	1/137
Newton's gravitational constant	G_N	6.673×10^{-11} m^{3}kg^{-1}s^{-2}
Fermi's decay constant	G_F	1.1×10^{-5} GeV^{-2}
conversion factor	$\hbar c$	197 MeV fm
conversion factor	eV	1.6×10^{-19} J