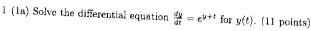
科目:應用數學(3001) 校系所組:中大照明與顯示科技研究所甲組、天文研究所

交大電子物理學系丙組、物理研究所

清大物理學系、先進光源科技學位學程物理組、天文研究所

陽明生醫光電研究所理工組A



- (1b) Find the general solution of $\frac{d^2y}{dx^2} \frac{4}{x}\frac{dy}{dx} + \frac{6}{x^2}y = \frac{4}{x^2}$. (11 points)
- (1c) A tank initially contains 40 g of salt mixed in 100 liters of water. A solution contains 4 g of salt per liter is pumped into the tank at a rate of 5 liter/min. The stirred mixture flows out the tank at the same rate. How much salt is in the tank after 20 minutes? (12 points)



- (a) Given a 2×2 hermitian matrix $B = \begin{pmatrix} 0 & \theta \\ \theta & 0 \end{pmatrix}$, and $x = \begin{pmatrix} a \\ b \end{pmatrix}$ as a column matrix, (i) solve the eigenvalue problem $Bx = \lambda x$ for the eigenvalues and eigenvectors, and then (ii) find a matrix S such that $B_s = SBS^{-1}$
- (b) Prove that $\det \exp[iA] = \exp[iTrA]$ for any $n \times n$ hermitian matrix A. Prove this result from the fact that

[Remark: $\det B$ and Tr B are the determinant and the trace of the matrix B respectively. θ , a and b are constant parameters. $\exp[A] = \sum_{k=0}^{\infty} [A]^k / k!$ defines the exponential mapping of any matrix A.

Define a $27(=3\times3\times3)$ components 3-index function A_{ijk} for all i,j,k=1,2,3 as a totally antisymmetric 3-index function with $A_{123}=1$. Here totally antisymmetric means that $A_{ijk}=-A_{jik}=-A_{ikj}$ for all

- (a) (i) List all non-vanishing components of A_{ijk} , e.g. $A_{132} = -1$, and \cdots from the antisymmetric properties of A_{ijk} . (ii) Also give a reason, from the symmetric properties of A_{ijk} , to explain why there are totally k(=?)
- (b) The determinant of any 3×3 matrix B can be defined as $\det B = \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 A_{ijk} \left[B_{1i} B_{2j} B_{3k} \right]$. From the symmetric properties of A_{ijk} , show that $A_{lmn} \det B = \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 A_{ijk} \left[B_{1i} B_{mj} B_{nk} \right]$.

[Remark: This definition agrees with the conventional definition of the determinant of any 3×3 matrix B that $\det B = \det \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{pmatrix} = B_{11}[B_{22}B_{33} - B_{23}B_{32}] - B_{21}[\cdots] + B_{31}[\cdots].$

4 Consider the function

$$f(z) = \frac{1}{1+z}.$$

- (a) Expand f(z) about the point z = 0. What is the convergence radius? (5 points)
- (b) Expand f(z) about the point z = i. What is the convergence radius? (5 points)
- 5 Perform the following integrals.
 - (a) For $\omega > 0$ and t is real, calculate

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{e^{-ikt}}{k^2 + \omega^2} dk. \quad (8 \text{ points})$$

(b) For $\sigma > 0$, calculate

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x \sin x}{x^2 - \sigma^2} dx. \quad (10 \text{ points})$$

(c) For $\sigma > 0$ and $\epsilon > 0$, calculate

$$\lim_{\epsilon \to 0} \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x \sin x}{x^2 - (\sigma - i\epsilon)^2} dx.$$
 (5 points)