所別：物理學系碩士班 不分組 科目：古典物理

You must show the procedures of your calculation, otherwise you won't get any credit.

1. A simple harmonic oscillator consists of a 100-g mass attached to a spring whose force constant is 10^4 dyne/cm. The mass is displaced 3 cm and released from the rest. Calculate (a) the natural frequency ν_0 and the period τ_0, (5%) (b) the total energy, (5%) and (c) the maximum speed. (5%).
2. Find the center of mass of a uniformly solid hemisphere of radius a. (10%)
3. Consider the case of the disk rolling down an inclined plane (see Figure 1). (a) Derive the Lagrangian L and the equation of constraint in terms of y and θ. (5%) (b) Write down the Lagrangian equations. (5%) (c) Calculate the force of constraint and the angular acceleration. (10%).
4. A circular ring of radius a carries uniform charge ρ_L C/m and is placed on the xy-plane with axis the same as the z-axis. (a) Show that $\vec{E}(0,0,h)=\frac{\rho_L a h}{2\varepsilon_0[h^2+a^2]^{3/2}}\vec{a}_z$, (10%) (b) What value of h gives the maximum value of $\vec{E}$? (5%). (c) If the total charge on the ring is Q, find $\vec{E}$ as $a\to 0$. (5%).
5. Conducting spherical shells with radii a = 10cm and the b =30cm are maintained at a potential difference of 100 V such that V (r = b) = 0 and V(r = a) =100. Determine V and **E** in the region between the shells (10%). If $\varepsilon_r = 2.5$ in the region, determine the total charge induced on the shells and the capacitance of the capacitor. ($\varepsilon_0 \cong \frac{10^{-9}}{36\pi} F/m$) (10%)
6. A conducting bar can slide freely over two conducting rails as shown in Figure 2. Calculate the induced voltage in the bar. (a) If the bar is stationed at y = 8cm and $\mathbf{B}= 4\cos 10^6 t\ \mathbf{a}_z$ mWb/m^2 (5%) (b) If the bar slides at a velocity $\mathbf{u} = 20\ \mathbf{a}_y$ m/s and $\mathbf{B} = 4\mathbf{a}_z$ mWb/m^2 (5%) (c) If the bar slides at a velocity $\mathbf{u} = 20\ \mathbf{a}_y$ m/s and $\mathbf{B}=4\cos(10^6 t-y)\ \mathbf{a}_z$ mWb/m^2. (5%) (Useful equations:

$V_{emf}=\oint \vec{E}\cdot d\vec{l}=-\int\frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}+\oint(\vec{u}\times\vec{B})\cdot d\vec{l}$ and $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}+\nabla\times(\vec{u}\times\vec{B})$)

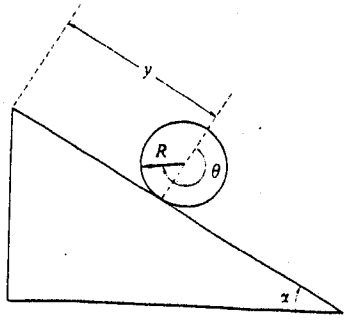

Fig. 1

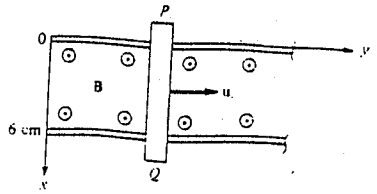

Fig. 2

參考用