所別：物理學系暨生物物理研究所碩士班　科目：近代物理

1. An X ray of wavelength 0.05 nm scatters from a gold target. (a) Can the x ray be Compton-scattered from an electron bound by as much as 62 keV? (b) What is the largest wavelength of scattered photon that can be observed? (c) What is the kinetic energy of the most energetic recoil electron and at what angle does it occur? (20%)
2. Consider a particle whose normalized wave function is $\psi(x) = 2\alpha\sqrt{\alpha}xe^{-\alpha x}$ for x>0 and $\psi(x) = 0$ for x< 0. (a) Write down its wave function in momentum space and use this to calculate $\langle p \rangle$ and. $\langle p^2 \rangle$. (b) Find the location of maximum possibility to find this particle. (c) What is the possibility that the particle is found between x=0 and x = 1/α? (20%)
3. A measurement of S_x for a spin 1/2 system leads to the eigenvalue $+\hbar/2$. Subseqently, a measurement of $S_x\cos\phi + S_x\sin\phi$ is carried out. What is the possibility that the result is $+\hbar/2$. (20%)
4. An electron in an oscillating electric field is described by the Hamiltonian operator $H = p^2/2m - (eE_0\cos\omega t)x$. Calculate expressions for the time dependence of $\langle x \rangle, \langle p \rangle$ and $\langle H \rangle$. (20%)
5. An electron in the Coulomb field of proton is in a state described by the wave function $\frac{1}{6}\left[4\psi_{100}(\vec{r}) + 3\psi_{211}(\vec{r}) - \psi_{210}(\vec{r}) + \sqrt{10}\psi_{21-1}(\vec{r})\right]$. (a) What is the expectation value of the energy? (b) What is the expectation value of L^2? (c) What is the expectation value of L_z? (20%)

$$Y_{0,0} = \frac{1}{\sqrt{4\pi}}$$

$$Y_{1,1} = -\sqrt{\frac{3}{8\pi}}\, e^{i\phi} \sin\theta$$

$$Y_{1,0} = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y_{2,2} = \sqrt{\frac{15}{32\pi}}\, e^{2i\phi} \sin^2\theta$$

$$Y_{2,1} = -\sqrt{\frac{15}{8\pi}}\, e^{i\phi} \sin\theta\cos\theta$$

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}}\,(3\cos^2\theta - 1)$$

$$R_{10}(r) = 2\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$$

$$R_{20}(r) = 2\left(\frac{Z}{2a_0}\right)^{3/2}\left(1 - \frac{Zr}{2a_0}\right) e^{-Zr/2a_0}$$

$$R_{21}(r) = \frac{1}{\sqrt{3}}\left(\frac{Z}{2a_0}\right)^{3/2}\frac{Zr}{a_0} e^{-Zr/2a_0}$$

$$R_{30}(r) = 2\left(\frac{Z}{3a_0}\right)^{3/2}\left[1 - \frac{2Zr}{3a_0} + \frac{2(Zr)^2}{27a_0^2}\right] e^{-Zr/3a_0}$$

$$R_{31}(r) = \frac{4\sqrt{2}}{3}\left(\frac{Z}{3a_0}\right)^{3/2}\frac{Zr}{a_0}\left(1 - \frac{Zr}{6a_0}\right) e^{-Zr/3a_0}$$

$$R_{32}(r) = \frac{2\sqrt{2}}{27\sqrt{5}}\left(\frac{Z}{3a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right)^2 e^{-Zr/3a_0}$$