

所別：物理學系碩士班 一般生 科目：近代物理
學位在職生

1. A particle of mass m is confined by an one-dimensional infinite square well potential, $V = \infty$ when $|x| > a$ and $V = 0$ when $|x| \leq a$.
- (a) By solving Schrödinger equation, derive the possible eigenenergies (eigenvalues of the Hamiltonian operator) and the corresponding normalised eigenfunctions. (10%)
- (b) We conducted an ideal measurement for the position of the particle and observed the particle at $x = a/4$. What do you expect to see if we do the position measurement again for this system? What do you expect to see if you try to observe the momentum of this particle? Why? (10%)
- (c) If the particle is in the state described by a wave function

$$\Psi(x) = x, \text{ when } |x| \leq a,$$
$$\Psi(x) = 0, \text{ when } |x| > a,$$

Find the relative probability that a measurement of the energy will give the results E_1 (the eigenenergy of the ground state) and E_3 (the eigenenergy of the second excited state). (15 %)

2. Write down the criterion enabling you to decide that Quantum Mechanics (or Classical Mechanics) should be used to describe an observed physical system, and explain why that is. (15%)
3. In the Stern-Gerlach experiment, atoms are guided to pass through a non-uniform magnetic field largely in one direction.
- (a) Why a beam of silver atoms in its ground state is split into two separate beams? (5%)
- (b) A beam of spin $1/2$ atoms goes through a series of such Stern-Gerlach measurements as follows:
- (1) The first measurement accepts $s_z = \hbar/2$ atoms and rejects $s_z = -\hbar/2$ atoms.
- (2) The second measurement accept $s_n = \hbar/2$ atoms and rejects $s_n = -\hbar/2$ atoms, where s_n is the eigenvalue of the operator $\mathbf{S} \cdot \mathbf{n}$, with \mathbf{n} making an angle β in the xz -plane with respect to the z -axis.
- (3) The third measurement accepts $s_z = -\hbar/2$ atoms and rejects $s_z = \hbar/2$ atoms.
- What is the intensity of the final $s_z = -\hbar/2$ beam when the $s_z = \hbar/2$ beam surviving the first measurement is normalised to unity? (10%)

注意：背面有試題

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4. Suppose that four non-interacting particles are placed in a three-dimensional harmonic oscillator potential, for which the single particle energy is

$$E = (N_x + N_y + N_z + 3/2) \hbar \omega.$$

What is the allowed lowest energy for the four-particle state when the particles are:

- (a) Distinguishable, spinless bosons; (5%)
 - (b) Identical, spinless bosons; (5%)
 - (c) Identical fermions each with spin $s = 5/2$? (5%)
5. (a) In 1905 Einstein published his *Electrodynamics of Moving Bodies*, in which he developed *the special theory of relativity* from two basic postulates. What are the two postulates? (10%)
- (b) Marry (at point A) and John's train (with proper length $L_0 = 250$ m) pass each other with constant relative speed v . Marry measures a time interval of $5.0 \mu\text{s}$ for the train to pass her. In term of c (light speed = 3×10^8 m/s), what is the relative speed v between Marry and John? (10%)