所別：物理學系碩士班 一般生 科目：應用數學
學位在職生

1. Given the function $f(x, y, z)$ as

$$f(x, y, z) = x^2 + \frac{y^2}{4} + \frac{z^2}{4}$$

and $\mathbf{A} = \nabla f$.

(a) (5) Make a graph showing the curves produced by $f(x, y, z) = 1$, $f(x, y, z) = 2$, and $f(x, y, z) = 3$ on the x–y plane. Draw $\mathbf{A}$ at $(1, 1, 0)$, $(2, -1, 0)$, and $(-1, -2, 0)$ on the graph too.

(b) (5) Calculate the line integral

$$\int \mathbf{A} \cdot d\mathbf{l}$$

with the integration path given by

$$(0, 0, 0) \rightarrow (1, 0, 0) \rightarrow (1, 1, 0) \rightarrow (1, 1, 1)$$

(c) (5) Calculate the surface integral

$$\int \mathbf{A} \cdot \hat{\mathbf{n}} ds$$

with the integration surface as the squre defined by the four points $(1, 0, 0)$, $(1, 1, 0)$, $(1, 1, 1)$, $(1, 0, 1)$.

(d) (5) Calculate the line integral

$$\int \mathbf{A} \times d\mathbf{l}$$

The the integration path as an arc on the x–y plane is defined as [with (r, θ) being the polar coordinate on the x–y plane]

$$(r = 1, \theta = 0) \rightarrow (r = 1, \theta = \pi/4) \rightarrow (r = 1, \theta = \pi/2)$$

2. (a) (6) Solve the differential equation

$$x^2 \frac{d^2y}{dx^2} - 2y = x$$

with the boundary conditions $y(x = 1) = 1$ and $y(x = 2) = 4$.

(b) (7) Find the general solution of

$$x\frac{dy}{dx} + (1 + x)y = e^x$$

(c) (7) Find the general solution of

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = \sin x$$

(Hint:Eliminate the first derivative term)

3. (a) (5) If $\delta(x)$ is the Dirac delta function, give an argument on why

$$\frac{d\delta(x)}{dx} = -\delta(x)/x$$

注意：背面有試題

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(b) (5) If a system has energy levels given by

$$E_n = n^2 \epsilon_0, \qquad n = 1, 2, 3, \ldots$$

What is the distribution of energy levels $D(E)$ (the number of energy levels per unit energy interval) ?

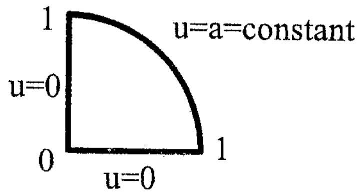
(c) (10) Transform the vector $(1, 2, 3)$ using the bases given by the eigenvectors of the matrix

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

4. (a) (10) Find the solution of

$$\nabla^2 f(\mathrm{r}) = 0$$

inside the following region, with the boundary condition as indicated.



(b) (10) Solve for $0 \le x \le L$,

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad u(0,t) = u(L,t) = 0, \quad u(x,0) = u_0$$

5. With z the complex variable

(a) (6) What is the curve represented by

$$\mathrm{Re}(1 - z) = |z|$$

(b) (7) Along which curves in the complex plane do the functions $\sinh z$ have real values?

(c) (7) Find

$$\int_{-\infty}^{+\infty} \frac{dx}{x^4 + a^4}$$