科目：工程數學 B(5004)　　校系所組：中大電機工程學系(電子組、固態組)
中大通訊工程學系(甲組)
交大電子研究所(甲組、乙組)
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參考用

1. (10%) Consider the matrix $\mathbf{A}=\begin{bmatrix}\mathbf{B} & \mathbf{C}\\ \mathbf{0} & \mathbf{D}\end{bmatrix}=\begin{bmatrix}0 & 1 & 3 & 0\\ * & * & 0 & 4\\ 0 & 0 & 6 & 1\\ 0 & 0 & 1 & 6\end{bmatrix}$.

(a) (2%) Indicate the 2 by 2 matrix **B** if the eigenvalues of **B** are 1 and 2.
(b) (4%) Find the eigenvalues of the 4 by 4 matrix **A**.
(c) (4%) Let $\mathbf{B}^5=\begin{bmatrix}1 & 1\\ x & y\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \lambda\end{bmatrix}\begin{bmatrix}1 & 1\\ x & y\end{bmatrix}^{-1}$, please find out the unknown entries x, y, and λ.

2. (10%) Assume $\mathbf{M}_{22}$ denotes the vector space consisting of all 2 by 2 matrices. That is, $\mathbf{M}_{22}=\{\begin{bmatrix}a & b\\ c & d\end{bmatrix}, \forall a, b, c, d \in \Re\}$. Given two matrices $\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$ and $\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}$ in $\mathbf{M}_{22}$, we define their inner product to be

$$\left\langle \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}, \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}\right\rangle \equiv a_{11}b_{11}+a_{12}b_{12}+a_{21}b_{21}+a_{22}b_{22}.$$

(a) Assume the set of four matrices $\{\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}, \begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}, \begin{bmatrix}1 & 1\\ -1 & 0\end{bmatrix}, \begin{bmatrix}-2 & 0\\ 4 & 4\end{bmatrix}\}$ span a subspace **K** in $\mathbf{M}_{22}$.

(i) (3%) Please find the dimension of **K.**
(ii) (3%) Please find the largest subspace in $\mathbf{M}_{22}$ that is orthogonal to **K**.

(b) (4%) Let T: $\mathbf{M}_{22}\rightarrow \mathbf{M}_{22}$ be a linear transformation defined as

$$T(\begin{bmatrix}a & b\\ c & d\end{bmatrix})=\begin{bmatrix}a+b-c+d & 2b+3c-d\\ -a+3c & 2a+c+d\end{bmatrix}$$ for any $\begin{bmatrix}a & b\\ c & d\end{bmatrix}\in\mathbf{M}_{22}$. Please find the null space of T.

3. (15%) Let X and Y be two independent random variables uniformly distributed over the interval [0, 1]. What is the probability that the quadratic equation $t^2+Xt+2Y=0$ has two real roots?

4. (10%) Let X_1, X_2, … be a sequence of i.i.d. Poisson random variables with parameter 1. Using the central limit theorem to calculate $\lim_{n\to\infty}\sum_{i=0}^{n} e^{-n}\frac{n^i}{i!}$.

注意：背面有試題

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參考用

5. (8%) An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume the plate shown in Fig. 1 represents a cross section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let T_1, T_2, T_3, and T_4 denote the temperatures at the four interior nodes of the mesh in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes. For instance, $T_1 = (10+30+T_2+T_3)/4$.
   (a) (3%) Write a system of four equations whose solution gives estimates for the temperature T_1, T_2, T_3, and T_4.
   (b) (5%) Solve the system of equations in (a).

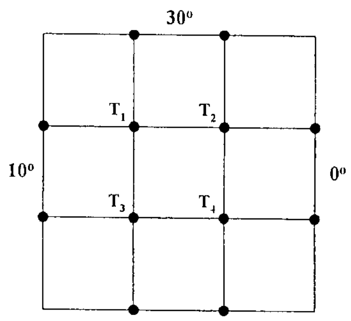

Fig. 1

6. (12%) Consider the function

$$F(x)=\begin{cases}0, & x<0\\ x+1/2, & 0\le x<1/2\\ 1, & x\ge 1/2.\end{cases}$$

If X is a random variable whose cumulative distribution function is given by $F(x)$, find (a) (3%) the probability of $X=0$, (b) (3%) the probability of $0\le X\le 1/4$, (c) (3%) the mean of X, and (d) (3%) the variance of X.

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7. (10%) Assume T: $\mathbf{R}^3 \to \mathbf{R}^2$ is a linear transformation with $T(\begin{bmatrix}1\\0\\0\end{bmatrix}) = \begin{bmatrix}1\\1\end{bmatrix}$, $T(\begin{bmatrix}1\\1\\0\end{bmatrix}) = \begin{bmatrix}1.5\\0.5\end{bmatrix}$, and $T(\begin{bmatrix}1\\1\\1\end{bmatrix}) = \begin{bmatrix}1.5\\1.5\end{bmatrix}$.

(a) (2%) If we represent any vector **x** in $\mathbf{R}^3$ as $\mathbf{x} = \begin{bmatrix}a\\b\\c\end{bmatrix}$ and the vector T(**x**) in $\mathbf{R}^2$ as $T(\mathbf{x}) = \begin{bmatrix}d\\e\end{bmatrix}$, please find the corresponding matrix **A** such that $\begin{bmatrix}d\\e\end{bmatrix} = \mathbf{A}\cdot\begin{bmatrix}a\\b\\c\end{bmatrix}$.

(b) (2%) Find the range of T.

(c) (6%) If we represent any vector **x** in $\mathbf{R}^3$ as $\mathbf{x} = x_1\cdot\begin{bmatrix}1\\0\\1\end{bmatrix} + x_2\cdot\begin{bmatrix}0\\1\\0\end{bmatrix} + x_3\cdot\begin{bmatrix}-1\\0\\1\end{bmatrix}$ and the vector T(**x**) in $\mathbf{R}^2$ as $\mathbf{T}(\mathbf{x}) = y_1\cdot\begin{bmatrix}1\\1\end{bmatrix} + y_2\cdot\begin{bmatrix}0\\1\end{bmatrix}$, please find the corresponding matrix **M** such that $\begin{bmatrix}y_1\\y_2\end{bmatrix} = \mathbf{M}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$.

8. (7%) Suppose a certain drug test is 99% sensitive and 99% specific, that is, the test will correctly identify a drug user as testing positive 99% of the time, and will correctly identify a non-user as testing negative 99% of the time. A corporation decides to test its employees for opium use, and 0.5% of the employees use the drug. Given a positive drug test, what is the probability that an employee is actually a drug user?

9. (6%) There are ten balls in a box numbered by 1 to 10. In each experiment, a ball is picked at random from the box and is NOT put back to the box. Find the average number of experiments required in order to obtain the ball number 2.

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10. (12%) Let **S** denote a set of zero-mean Gaussian random variables and we can treat **S** as a linear vector space. The inner product between two random variables U and V in **S** is defined as $\langle U,V\rangle \equiv E[U\,V]$. That is, the inner product of two random variables equals to the cross correlation between them. Given three zero-mean Gaussian random variables X, Y, and Z, with variances $\sigma^2_X = \sigma^2_Y = \sigma^2_Z = 2$ and $E[X\,Y] = E[Y\,Z] = E[Z\,X] = 1$, please answer the following questions.

(a) (2%) From the set of four vectors $\{(X-Y), Z, (-X+Y), (X+Z)\}$, we can choose three linearly independent vectors. Write down one possible choice.

(b) (3%) Continue with (a). In your choice, which two vectors are orthogonal to each other?

(c) (7%) Continue with (b). Start with the two orthogonal vectors, please use the Gram-Schmidt process to build a set of three orthonormal vectors.