※請在答案卷內作答

1. (5%) Find the general solution for $\frac{d^2y}{dx^2}+y=2cosx$

2. (5%) Find the Laplace transform for $f(t)=e^{2t}cost$

3. (5%) Find the Laplace transform for $f(t)=t-[t]$, where $[t]$ is the largest integer that is not larger than $t$.

4. (5%) Find the inverse Laplace transform for $\frac{1}{s^3+1}$ .

5. (5%) $A=\begin{pmatrix} 4 & -5 \\ 2 & -3 \end{pmatrix}$，find $e^{At}$ by using Cayley-Hamilton theorem.

參考用

6. (6%) Evaluate the following integrals.

(a) (2%) $\int_0^i z^2\,dz$

(b) (2%) $\oint_C e^z\,dz$, where C: $|Z|=1$, clockwise.

(c) (2%) $\oint_C \frac{1}{z-1}dz$, where C: $|Z|=5$, clockwise.

7. (6%)

$$f(z)=\frac{1}{(z-1)(z-2)}$$

Integrate $f(z)$ counter clockwise, $\oint_C f(z)\,dz$, around $C_1$, $C_2$, and $C_3$.

(a) (2%) $C_1: |z|=\frac{1}{2}$

(b) (2%) $C_2: |z|=\frac{3}{2}$

(c) (2%) $C_3: |z|=\frac{5}{2}$

注意:背面有試題

**8.** (6%)

$z = x + jy$

$f(z) = z^2$ , Calculate $f'(z)$ and $g'(z)$.

$g(z) = |z|^2$

**9.** (7%)

$f(z) = z^2 e^{\frac{1}{z}}$

find the Laurent Series of $f(z)$.

**10.** (10%)

$$A = \begin{pmatrix} 8 & -6 \\ 4 & -2 \end{pmatrix}$$

(a) (5%) $Ax = \lambda x$ , $\lambda$ is the eigenvalue of $A$, and $x$ is the eigenvalue of $A$, Find $\lambda$ and $x$.

(b) (5%) $A = SDS^{-1}$, $S$ is the matrix with eigenvectors of $A$, and $D$ is a diagonal matrix which is composed by the eigenvalues of $A$. Find $S$, $D$, and $S^{-1}$.

**11.** (15%) Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

(a) (3%) Find $rank(A)$.

(b) (4%) Find $N(A)$.
($N(A)$ is the nullity of $A$.)

(c) (4%) Find $dim(N(A)) + rank(A)$
($dim(N(A))$ is the dimension of $N(A)$)

(d) (4%) $Ax = b$, find $x$.

**12.** (12 %) Let $V$ be a finite-dimensional vector space and $T: V \to V$ be linear. Suppose $rank(T) = rank(T^2)$. Prove that $R(T) \cap N(T) = \{0\}$.

注意:背面有試題

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13. (13%) Given a vector space $V$ over $F$. Define the dual space of $V^* \times V^*$ as the set of all function (also known as linear functional) from $V$ to $F$, i.e., $V^* \stackrel{\text{def}}{=} \{f | f : V \to F\}$. It is obvious that $V^*$ is itself also a vector space with the addition $+ : V^* \times V^* \to V^*$ and scalar multiplication $* : F \times V^* \to V^*$ define as pointwise addition as well as pointwise scalar multiplication. Given any linear transformation $T : V \to W$. The transpose $T^t$ is a linear transformation from $W^*$ to $V^*$ defined by $T^t(f) = fT$ for any $f \in W^*$. For every subset S of V, we define the annihilator $S^0$ as $S^0 \stackrel{\text{def}}{=} \{f \in V^* | f(x) = 0 \ \forall x \in S\}$. Suppose $V, W$ are both finite-dimensional vector spaces and $T : V \to W$ is linear. Prove that $N(T^t) = (R(T))^0$.