類組：電機類　科目：訊號與系統(300B)　　　　共 3 頁 第 1 頁

計算題應詳列計算過程，無計算過程者不予計分

1. (10%) Consider the cascade of two linear time-invariant (LTI) systems as in Figure 1, where in this case $h_1[n] = \sin(8n)$ and $h_2[n] = a^n u[n], |a| < 1$ and where the input is $x[n] = \delta[n] - a\delta[n-1]$.
   (a) (5%) Compute the convolution of $x[n]$ and $h_2[n]$
   (b) (5%) Determine the output y[n]

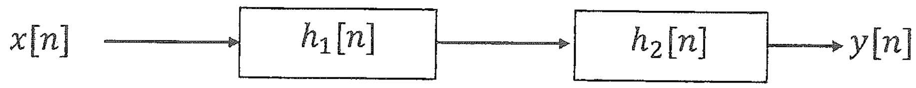

Figure 1

2. (15%) Consider an LTI system:
   (a) (8%) Consider an LTI system with frequency response

$$H(e^{j\omega}) = \frac{1-e^{-j2\omega}}{1+\frac{1}{2}e^{-j4\omega}} \quad -\pi < \omega < \pi \ ,$$

   Please determine the output $y[n]$, if the input $x[n] = \sin(\frac{\pi n}{4})$.

   (b) (7%) Consider an LTI system with impulse response $h(t) = e^{-5t}u(t)$, if the output of the system is $y(t) = e^{-3t}u(t) - e^{-5t}u(t)$, please determine the input $x(t)$ that produce this output.

3. (15%) There is a periodic signal that is real. Given its Fourier series coefficients a_k where k is an integer, and $a_{-1} = 2,\ a_0 = 1,\ a_2 = 3$, and $a_k = 0$ for k > 2,
   (a) (5%) Is this signal a continuous or discrete-time signal? Justify your answer.
   (b) (5%) Find its time-averaged power over one fundamental period.
   (c) (5%) Assume its fundamental period is M seconds, plot its Fourier-transform spectrum.

4. (8%) A periodic signal $x(t)$ is given, which is defined as follows. Please find its fundamental angular frequency.

$$x(t) = \int_{-\infty}^{\infty} \frac{\cos(5\pi(t-\tau))\sin(10\pi\tau)}{\pi\tau} d\tau$$

5. (7%) Given a signal $x(t)$ which is defined as follows. Please plot its Fourier transform spectrum $X(j\omega)$.

$$x(t) = \frac{sin^2(10\pi t)}{\pi t^2}$$

(Hint: $x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$ and $X(j\omega) = \int_{-T_1}^{T_1} e^{-j\omega t} dt = 2\frac{\sin \omega T_1}{\omega}$)

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6. (5%) Determine whether each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.
   (a) (2%) $x[n] = \sum_{k=-\infty}^{\infty}\{\delta[n-2k] - \delta[n-1-3k]\}$
   (b) (3%) $y[n] = e^{j3\pi n/5} - e^{j2n/5}$

7. (15%) The overall system for filtering a continuous-time signal using a discrete-time filter is shown in Figure 2(a). $X_c(j\omega)$ and $H(e^{j\omega})$ are given in Figure 2(b), with the sampling period T = 1/20000 sec, and $p(t) = \sum_{n=-\infty}^{\infty}\delta(t-nT)$.
   (a) (10%) Sketch $X_p(j\omega)$, $X(e^{j\omega})$, $Y(e^{j\omega})$, $Y_p(j\omega)$, and $Y_c(j\omega)$.
   (b) (5%) The overall system, with input $x_c(t)$ and output $y_c(t)$, is equivalent to a continuous-time lowpass filter with frequency response $H_{eff}(j\omega)$. Sketch $H_{eff}(j\omega)$ and determine the relationship between the continuous-time impulse response $h_{eff}(t)$ and the discrete-time impulse response $h[n]$.

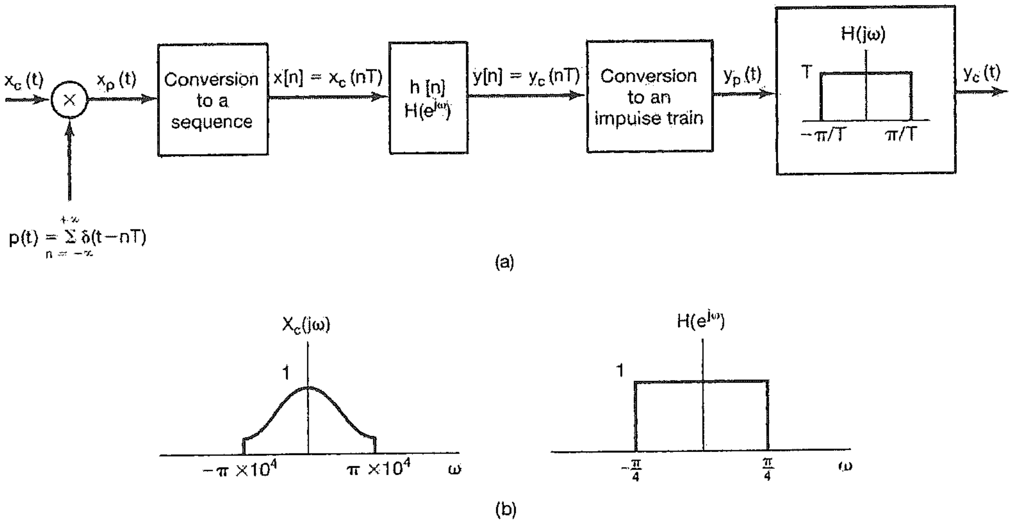

Figure 2

8. (10%) Consider a continuous-time LTI system with input $x(t)$, output $y(t)$, and impulse response $h(t)$.

$$x(t) = 0, t > 0 \quad \text{and} \quad X(s) = \frac{s+2}{s-2}$$

$$y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$$

where $X(s)$ is the Laplace transform of $x(t)$ and $u(t)$ is the step function.
   (a) (5%) Determine the transform function $H(s)$ of the system and the region of convergence (ROC).
   (b) (2%) Determine $h(t)$
   (c) (3%) Determine the output $y(t)$ given the input $x(t) = e^{-2t}$ for $-\infty < t < \infty$

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9. (15%) Use the following four clues to determine the discrete-time signal $x[n]$ and its z-Transform $X(z)$.

   (i) Two poles at $z = 1/2$ and $z = -2$

   (ii) The signal is stable

   (iii) $x[0] = 1$

   (iv) The inverse of $X(z)$ has the pole at $z = 0$ and $z = 1$.

   (a) (10%) Determine $X(z)$ and its ROC.

   (b) (5%) Determine $x[n]$.