類組：電機類　科目：控制系統(300D)　

※請在答案卷內作答

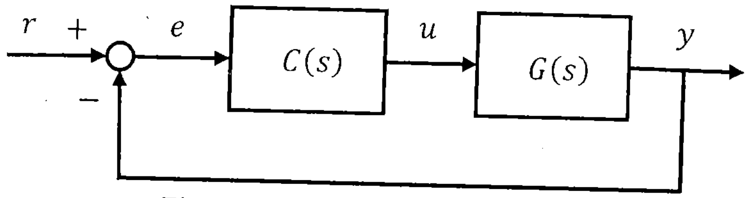

Figure 1: Feedback Control System

1. (25%)Consider the feedback control system in Figure 1. Suppose that

$$G(s) = \frac{1}{s(s+1)}, \quad \text{and } C(s) = \frac{K(s+b)}{s+a}$$

where $a, b, K > 0$. Answer the following questions.

(a) (5%)Find the range of K (in terms of a and b) in which the closed-loop system is stable.

(b) (10%)Let $b = 2(a+1)$. Find K (in terms of a) such that the closed-loop system has two pure imaginary poles. What are these pure imaginary poles (in terms of a)?

(c) (5%)Let $b = 0.5$. Find the numerical range of K such that the steady-state error of the closed-loop system with respect to the parabolic input, i.e. $r(t) = t^2$ for $t \geq 0$, is less than 0.01.

(d) (5%)Find the values of a, b and K such that the closed-loop transfer function from r to y is a second-order system with damping ratio $\xi = 0.5$ and natural frequency $\omega_n = 4$.

2. (25%)Consider the feedback control system in Figure 1. Suppose that

$$G(s) = \frac{s^2 + 3s + 2.5}{s^2(s^2 + 3s + 2)}$$

Answer the following questions.

(a) (5%)Let $C(s) = K$. Show that the closed-loop system is unstable for any K.

(b) (10%)Let $C(s) = K$. Sketch the root locus of the system for $K > 0$. You need to specify the intersection and angles of the asymptotes, the departure angles of the loci starting from the poles at $s = 0$, and the arrival angles of the loci entering the open-loop zeros.

(c) (10%)Let $C(s) = \frac{K(s+b)}{s+a}$. Find the numerical values of a and b such that there exists positive gain K that stabilizes the closed-loop system. Sketch the root locus to verify your design.

注意：背面有試題

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3. (25%) Let the state equations of an LTI system be represented by

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t),$$
$$y(t) = Cx(t)$$

where $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$.

(a) (10%) Find the state transition matrix.

(b) (6%) With the initial condition=0 and $\beta = 1$, choose B such that the transient response of $y(t)$ has only the component of te^{-2t}.

(c) (9%) With $B = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$ and $u(t)$, a unit-step input, choose the initial condition $x(0) = \begin{bmatrix} x_1(0) & x_2(0) & x_3(0) \end{bmatrix}^T$ such that $x(t)$ does NOT contain any transient parts, i.e., no e^{-t}, e^{-2t}, and te^{-2t}.

4. (25%) A unity-feedback system with the open loop transfer function $L(s) = \frac{K(s-z)}{s(s+p)}$, where

$L(j\omega) = \frac{K(p+z)}{\omega^2 + p^2} + j\frac{K(pz - \omega^2)}{\omega(\omega^2 + p^2)}$, and p, and z are all positive

(a) (10%) Sketch the Nyquist plot for $K>0$.

(b) (5%) Find the stability region of K and the oscillation frequency if exists from Nyquist plot.

(c) (10%) Let $p = z = 1$ and $|K| = 1/\sqrt{3}$. Find the Gain margin and Phase Margin and the corresponding gain crossover frequency and phase crossover frequency, if available.