1. (30%) Consider the following closed-loop characteristic polynomial

$$\chi(s) = s^4 + 2s^3 + K_1 s^2 + 4K_2 s + 2K_2(K_1 - 2)$$

where K_1, $K_2 > 0$.

(a) (6%) Find the conditions on K_1 and K_2 such that the closed-loop system is stable.

(b) (8%) Find all possible values of K_1 and K_2 such that the closed-loop system has a pair of pure imaginary poles with nonzero imaginary parts. What are these pure imaginary poles?

(c) (10%) Let $K_1 = 4$. Sketch the root locus as K_2 changes from 0 to ∞. You need to indicate the intersection of asymptotes, and the intersections of the root locus with the imaginary axis. You also have to calculate the departure angles of the complex poles.

(d) (6%) In what conditions does the closed-loop system have exactly three poles on the right-half plane?

2. (20%) Consider the feedback control system in Figure 1. Let $G(s) = \frac{s+1}{s(s+2)}$, and $C(s) = \frac{K}{\tau s+1}$. Answer the following questions.

(a) (5%) For unit ramp input, if the steady state error is 0.01, find K.

(b) (5%) Let K be determined in part (a). Sketch the root locus as τ changes from 0 to ∞.

(c) (5%) Let $\tau = 1$. Find K such that the percent maximum overshoot of the step response of the closed-loop system is 10%.

(d) (5%) Let $\tau = 0$ and $K = 2$. Find the gain crossover frequency and the phase margin.

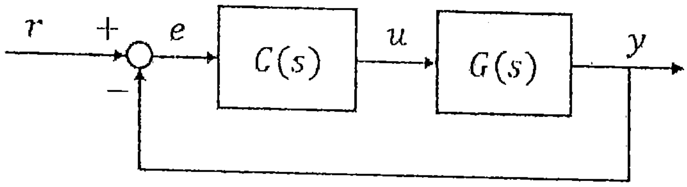
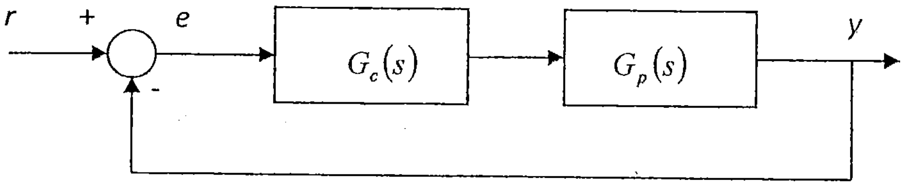


Figure 1 The feedback control system in Problem 2

3. (50%) Consider the following feedback system with plant $G_p(s)$ and controller $G_c(s)$.



Let $G_p(s)=\dfrac{s^2+2s+2}{s(s+1)}$, where $G_p(j\omega)=\dfrac{\omega^2}{\omega^2+1}-j\dfrac{\omega^2+2}{\omega(\omega^2+1)}$.

You are asked to design a P controller $G_c(s)=K$.

(a) (8%) Sketch the root locus as detail as possible and show that part of the root locus is a circle.

(b) (6%) Sketch the Nyquist plot for $K>0$.

(c) (5%) Analyze the stability from Nyquist plot for all K.

(d) (3%) For $K>0$, what are the Gain margin?

(e) (3%) In (e) and $r(t)=1$, what is the steady state output?

(f) (3%) For $K<0$, what is the Gain margin?

(g) (3%) When $K=-0.5$ and $r(t)=\cos(2t)$, what is the steady state output you expect?

(h) (6%) For $K>0$, if the gain crossover frequency is 1 rad/sec, what are the corresponding K and Phase margin? Hint: $\tan^{-1}(2)=1.11$ radian or $63.44°$; $\tan^{-1}(1)=0.79$ radian or $45°$

(i) (3%) What is the maximum allowable delay for part (h)?

(j) (6%) Formulate the system by Controllability Canonical Form. Let the state equation be $\begin{matrix}\dot{x}=\mathrm{A}x+\mathrm{B}e\\ y=\mathrm{C}x+\mathrm{D}e\end{matrix}$, where $x=[x_1 \quad x_2]^T$, $y=x_1$ and $x_2=\dot{x}_1$. Find the corresponding A, B, C and D in terms of K.

(k) (4%) Choose the state feedback controller $e=-Fx+r$, where $F=[f_1 \quad f_2]$, to replace the P controller so let K=1. If the closed loop poles are designed to have the damping ratio of $1/\sqrt{2}$ and the undamped frequency of $2\sqrt{2}$, find the controller.