

國立中央大學九十學年度轉學生入學試題

一、資電、地科學院 二年級 科目： 微積分 共 / 頁 第 /

壹. 填充題, 每題 10 分 (1 至 7 題為填充題僅須寫出答案, 不須計算過程.)

1 Evaluate the given integrals.

(a) $\int_0^1 \sin^{-1} x dx = \underline{\hspace{2cm}}$. (b) $\int_0^\infty te^{-t/3} dt = \underline{\hspace{2cm}}$.

2 (a) If $\lim_{x \rightarrow \infty} (\sqrt{x^2 + \alpha x + 1} - (x + 1)) = c$, then $\alpha = \underline{\hspace{2cm}}$.

(b) If $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} f'(x) = 0$ and $\lim_{x \rightarrow 0^+} f''(x) = a$, then $\lim_{x \rightarrow 0^+} xf(t/\sqrt{x}) = \underline{\hspace{2cm}}$.

3 If the function f is defined by $f(x, y) = -xye^{-(x^2+y^2)/2}$.

(a) The saddle point of f is $\underline{\hspace{2cm}}$. (b) The local minimum of f is $\underline{\hspace{2cm}}$.

4 Perform the following differentiations.

(a) $\frac{d}{dx} [\pi^x x^\pi] = \underline{\hspace{2cm}}$. (b) $\frac{d}{dx} \left(\int_0^{\ln x} \frac{dt}{\sqrt{4+e^t}} \right) = \underline{\hspace{2cm}}$.

5 Evaluate the following integrals.

(a) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2} dy dx = \underline{\hspace{2cm}}$.

(b) $\int_C 2xy dx + (x^2 + y^2) dy = \underline{\hspace{2cm}}$,

where C is the circular arc given by $x = \cos t$, $y = \sin t$ ($0 \leq t \leq \pi/2$).

6 (a) The interval of the convergence of the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$ is $\underline{\hspace{2cm}}$.

(b) Let n be the degree of the Taylor polynomial centered at 1 required to approximate $f(x) = e^x$, $x \in [0, 2]$ to an accuracy of ± 0.001 , then $n = \underline{\hspace{2cm}}$.

7 (a) A spherical balloon is expanding. If the radius is increasing at the rate of 2 inches per minute, at what rate is the volume increasing when the radius is 5 inches? $\underline{\hspace{2cm}}$.

(b) Let V be the dimensions of the rectangular package of largest volume subject to the sum of the length and the girth(腰圍) cannot exceed 108 inches, then $V = \underline{\hspace{2cm}}$.

貳. 計算及證明題, 每題 10 分 (8 至 10 題為計算及證明題, 須詳述計算過程.)

8 True or false? Justify your answers.

(a) If $f(x) < g(x)$ then $\lim_{x \rightarrow c} f(x) < \lim_{x \rightarrow c} g(x)$.

(b) $\lim_{x \rightarrow c} [f(x) + g(x)]$ can exist even if $\lim_{x \rightarrow c} f(x)$ and $\lim_{x \rightarrow c} g(x)$ do not exist.

9 Sketch the graph of the function $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3}$.

10 (a) Let $f(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0. \end{cases}$ Show that $f(x)$ is continuous at 0.

(b) Is $f(x)$ differentiable at 0?

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