參考用

1. Perform the following differentiations:

   (a) (5%) Let $F(x) = \int_1^{x^2} e^{-1/t^2} dt$ for $x \neq 0$. Find $F'(x)$.

   (b) (15%) Let

   $$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   Find $f'(0)$ and $f''(0)$. (Give the details of computations).

2. (15%) Compute the integral

   $$\int \frac{dx}{x^4 + 2x^2 - 3}.$$

3. (15%) Find the volume of the solid bounded by the surface $x^2 + y^2 + z = 1$ and the coordinate xy plane.

4. (10%) Let $f(x, y) = x^2 - 2xy + 3y^2 + x - 5y + 1$. Find the extremum value or the saddle point of f, if any.

5. (10%) Let $\mathbf{f}(t) = (f_1(t), f_2(t), f_3(t))$ be a vector-valued function for $t \in [a, b]$. If $\mathbf{f}$ is continuous on $[a, b]$ and is differentiable on (a, b), prove that there is $\xi \in (a, b)$ such that the tangent vector of $\mathbf{f}$ at ξ is orthogonal to the vector $\mathbf{f}(a) \times \mathbf{f}(b)$.

6. Consider the power series $\sum_{n=1}^{\infty}(n+1)x^n$.

   (a) (5%) Find the radius R of convergence of this series.

   (b) (15%) Find the sum function $f(x)$ of this series for $|x| < R$.

7. (10%) Let $\{a_n\}_{n=1}^{\infty}$ be a decreasing sequence of positive terms with $\lim_{n\to\infty} a_n = 0$. Let f be the function defined for $x \in [0, \infty)$ by

   $$f(x) = (-1)^{n-1} a_n \quad \text{for} \quad x \in [n-1, n),\ n = 1, 2, 3, \ldots$$

   Prove that the improper integral $\int_0^{\infty} f(x)\, dx$ converges.