

Please show the details of your work for all questions. (two pages)

1. (10%) Find the general solution of  $y' + ky = e^{2kx}$ ,  $k \neq 0$ .

2. (10%) solve the initial value problem.

$$y'' - 2y' - 3y = 0, \quad y(0) = 2, \quad y'(0) = 14.$$

3. (10%) Find a general solution.

$$y'' + 144y = 24 \cos 12x$$

4. (10%) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -4.0 & 4.0 \\ -1.6 & 1.2 \end{bmatrix}$$

5. (5%) (a) Apply Rodrigues's formula  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$  to derive  $P_0(x)$ ,

$$P_1(x), P_2(x), P_3(x).$$

(5%) (b) Find the Fourier-Legendre series of  $f(x) = x^3 - x^2 + x - 1$ .

6. (10%) Apply Laplace Transform to solve

$$y'' - y = t, \quad y(0) = 1, \quad y'(0) = 1.$$

7. (10%) Using separation of variables to solve  $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ .

The boundary conditions are  $u(0, t) = 0$  and  $u(L, t) = 0$  for all  $t$ .

The initial conditions are  $u(x, 0) = f(x)$  and  $\left. \frac{\partial u(x, t)}{\partial t} \right|_{t=0} = 0$ .

Make sure to discuss the cases with (i)  $k = 0$ , (ii)  $k > 0$ , and (iii)  $k < 0$ .

參考用

注意：背面有試題

8. (10%) Find the Fourier series of the function

$$f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases} \quad P = 2L = 4, \quad L = 2$$

9. (10%) Find the arc length of  $y = x^{\frac{3}{2}}$  from  $x=0$  to  $x=4$ .

10. (10%) Write Laplacian  $\nabla^2$  in cylindrical and spherical coordinates, respectively.

參考用