1. The nonhomogeneous equation $y' + p(x)y = r(x)$ has the general solution in the following form of an integral

$$y(x) = e^{-h(x)}\left[\int e^{h(x)} r(x)dx + c\right]$$

where $h(x) = \int p(x)dx$. Please show this solution formula! (20%)

2. The thickness of a bottomset bed at the foot of a delta can often be well approximated by the expression

$$t = t_0 \exp(-x/x_0) \qquad\qquad \text{(EQ.1)}$$

where t is thickness, x is distance from the bottomset bed start and t_0 and x_0 are constants.

(a) Imagine approximating this sedimentary bed in cross-section by a series of rectangles of thickness t_i and width Δx (Fig. 1). What is the area of each rectangle? (5%)

(b) Now write down an approximate sum for the cross-sectional area of the entire bottomset bed with a series of N rectangles of equal width Δx but different thickness. (5%)

(c) By considering the limiting case of an infinite number of infinitesimally wide rectangles, write down and evaluate an integral equation giving the total cross-sectional area. (10%)

(d) If the present-day rate of sediment supply is 10 m^2/year and $x_0 = 5$ km and $t_0 = 1$ m, estimate the time taken to form the bed assuming the sediment supply rate has not altered through time. (10%)
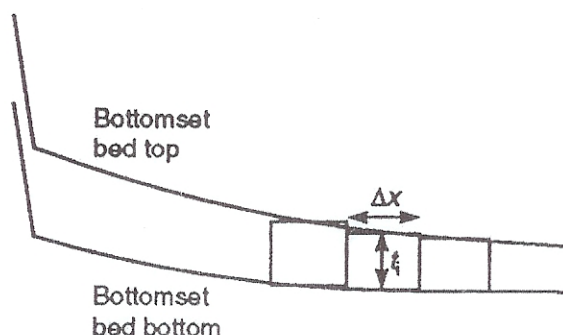

Figure 1

3. Considering that one-dimensional motion of a particle is governed by the following nonlinear differential equation:

$$\dot{x} = \sin x \qquad\qquad\qquad\qquad (EQ.2)$$

where x is the position of this particle and $\dot{x}$ is the time-derivative of its position, i.e. the velocity.

(a) Suppose that $x(t=0) = x_0$, please find the position function $x(t)$ for this particle. (10%) (Hint:

$\int \csc u\, du = -\ln|\csc u + \cot u| + C$ and $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$)

(b) Assuming $x_0 = \pi/4$, please plot the solution $x(t)$ qualitatively. What happens as $t \rightarrow \infty$? (10%)

(c) Now, let's consider another thinking way for (EQ.2). Please plot (EQ.2) in the so-called phase plane, that is the position as the abscissa and the velocity as the ordinate or the x-$\dot{x}$ plane, and draw the moving directions of this imaginary particle everywhere by arrows. (10%)

(d) For an arbitrary initial condition x_0, what is the behavior of $x(t)$ as $t \rightarrow \infty$? (10%) (Hint: Consider those points $x = n\pi$, n is an integer)

(e) For any one-dimensional system $\dot{x} = f(x)$ as shown in Fig. 2, could you describe the general behavior of the particle motion qualitatively as the evolution of time? (10%)
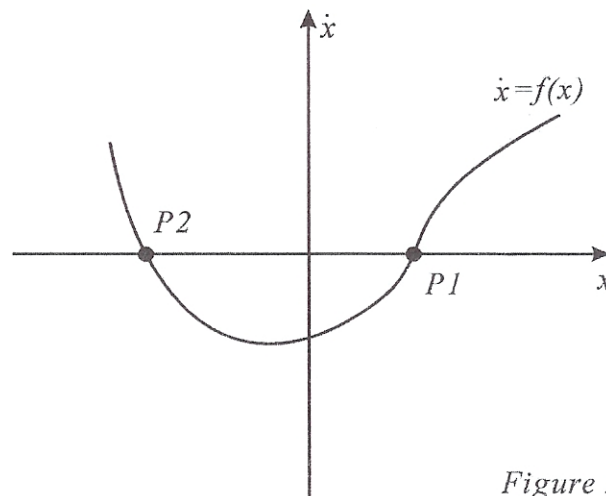

Figure 2