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1. Solve the following initial value problem  
 $y' = 1 + y^2, \quad y(0) = 0$   
(15%)
2. Solve the following initial value problem  
 $y'' + y' = 2 + 2x + x^2, \quad y(0) = 8, \quad y'(0) = -1$   
(15%)
3. Find eigenvalues and eigenfunctions of the following problem  
 $(x^{-1}y')' + (\lambda + 1)x^{-3}y = 0, \quad y(1) = 0, \quad y(e) = 0$   
(15%)
4. Using Laplace transforms, solve the following integral equation  

$$y = 2t - 4 \int_0^t y(\tau)(t - \tau) d\tau$$
(15%)
5. Diagonalize the following matrix.  

$$\begin{bmatrix} 5 & 10 & -10 \\ 10 & 5 & -20 \\ 5 & -5 & -10 \end{bmatrix}$$
(10%)
6. Evaluate the following surface integral  $\iint_S \mathbf{F} \cdot \mathbf{n} dA$ , where  
 $\mathbf{F} = [x = z, y + z, x + y]$  and  $S$  is the sphere of  $x^2 + y^2 + z^2 = 9$   
(10%)
7. Find the complex Fourier series of the following function  
 $f(x) = x^2, \quad -\pi < x < \pi$   
(10%)
8. Find solution  $u(x, y)$  of the following equation  
 $xu_{xy} + 2yu = 0$   
(10%)