甲、填充題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲、）部分，共 8 題，命題型態為填充題，不需詳列計算過程，僅將答案依題號順序依序寫在答案卷第一頁即可。若答案被包含在演算過程，無法挑出給予計分。

1. Find the average value of $g(x) = |x| - 1$ on $[-1, 3]$. Answer :_____

2. Find $\int x \sec^2 x \, dx$. Answer :_____

3. Evaluate $\int_0^2 \int_{y/2}^1 y e^{x^3} \, dx dy$. Answer :_____

4. Find the values of a and b that makes the function $f(x) = \begin{cases} \dfrac{2\sin^2 x}{x}, & \text{if } x > 0 \\ ax + b\cos x, & \text{if } x \leq 0 \end{cases}$ differentiable at $x = 0$. Answer :_____

5. Find the tangent line to the curve $x^2 \cos^2 y - \sin y = 0$ at $(0, \pi)$. Answer :_____

6. Find the volume of the solid obtained by revolving the region bounded by the curves $y = -x^2 + 4x$ and $y = x^2$ about the x-axis. Answer :_____

7. Evaluate $\lim_{x \to 0^+} (\sin x)^x$. Answer :_____

8. Find the sum of the series $\sum_{n=3}^{\infty} \dfrac{\ln(1 + \frac{1}{n})}{(\ln n)\ln(n+1)}$. Answer :_____

注意：背面有試題

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. An open rectangular box is to be constructed from material that costs \$3/ft^2 for the bottom and \$1/ft^2 for its sides. Find the dimensions of the box of greatest volume that can be constructed for \$36.

2. Use the limit definition to show that $g'(0)$ exists but$g'(0) \neq \lim_{x\to 0} g'(x)$, where

$$g(x) = \begin{cases} x^2 \sin\dfrac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

3. Determine if the series converges or diverges.

a. (6 分) $\displaystyle\sum_{n=0}^{\infty} e^{-n^2}$ **b.** (6 分) $\displaystyle\sum_{n=1}^{\infty} \sin\frac{1}{n}$

注意：背面有試題