甲、簡答題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲、）部分，共 8 題，命題型態為簡答題，不必詳列計算過程，倘若答案被包含在演算過程，將被視為試算流程，不予另行挑出計分。

1. Find the value of $\displaystyle\lim_{x\to\infty}\frac{x+\cos x}{x-\cos x}$. Answer :_____

2. Find all horizontal asymptotes of graph of the function $f(x)=\dfrac{|x|x}{x^2+1}$.
Answer :_____

3. Find the smallest positive $(x>0)$ inflection point of $F(x)=\displaystyle\int_0^x \cos(t^{3/2})\,dt$.
Answer :_____

4. A building in the shape of a rectangular box is to have a volume of 12,000 ft³. It is estimated that the annual heating and cooling costs will be \$2/square foot for the top, \$4/square foot for the front and back, and \$3/square foot for the sides. What is the minimal annual heating and cooling cost?
Answer :_____

5. Find the values of p for which the function $f(x,y)=\begin{cases}\dfrac{(xy)^p}{x^4+y^4}, & \text{if } (x,y)\neq(0,0)\\ 0, & \text{if } (x,y)=(0,0)\end{cases}$ is discontinuous at $(0,0)$.
Answer :_____

6. Evaluate $\displaystyle\int_0^{\ln 5}\int_{e^x}^5 \frac{1}{\ln y}\,dy\,dx$. Answer :_____

7. How many local extreme values does the function $f(x,y)=10xye^{-(x^2+y^2)}$ have?
Answer :_____

8. Let $f(x,y)=kxye^{-(x^2+y^2)}$ be a joint probability density function on $D=\{0<x<\infty, 0<y<\infty\}$, then $k=?$
Answer :_____

注意:背面有試題

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Goods 1 and 2 are available at prices (in dollars) of p_1 per unit of good 1 and p_2 per unit of good 2. A utility function $U(x_1, x_2)$ is a function representing the **utility** or benefit fo consuming x_j units of good j. The **marginal utility** of the jth good is $\partial U/\partial x_j$, the rate of increase in utility per unit increase in the jth good. Prove the following law of economics: Given a budget of L dollars, utility is maximized at the consumption level (a, b) where the ratio of marginal utility is equal to the ratio of prices:

$$\frac{\text{Marginal utility of good 1}}{\text{Marginal utility of good 2}} = \frac{\partial U/\partial x_1}{\partial U/\partial x_2} = \frac{p_1}{p_2}$$

2.

a. Determine whether the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n \ln\left(1 + \frac{1}{n}\right)$ diverges or converges conditionally or converges absolutely and give reasons for your answer. (6 points)

b. Show that if $\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3 + \sin(a_n)}{5}\right)^n$ converges. (6 points)

3. A trough with a trapezoidal cross section is to be constructed with a 1-foot base and sides that are 20 feet long and 1 foot wide, as shown in the figure. Only the angle θ can be varied. What value of θ will maximize the trough's volume?