科目 __工程數學__ 類組別____033,034,035____ 共 _1_ 頁第 _1_ 頁 *請在試卷答案卷（卡）內作答

10%   **1.**   Solve $(y^2 + yx)dx - x^2 dy = 0$.

10%   **2.**   Solve $3x^2 y'' + 6xy' + y = 0$.

10%   **3.**   Solve $f(t) + \int_0^t (t-\tau)f(\tau)d\tau = t$ for $f(t)$.

10%   **4.**   Determine whether the matrix $A = \begin{pmatrix} -5 & 9 \\ -6 & 10 \end{pmatrix}$ is diagonalizable. If so, find the matrix P that diagonalizes A and the diagonal matrix D such that $D = P^{-1}AP$.

10%   **5.**   Evaluate $\oint_C zdx + xdy + ydz$, where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$. Orient C counterclockwise as viewed from above.

10%   **6.**   Show that the set of functions $\{1, \cos nx\}$, $n = 1,2,3,\cdots$ is orthogonal on the interval $[-\pi, \pi]$.

10%   **7.**   Find the Fourier Series of $f(x) = \begin{cases} 1 & -1 < x < 0 \\ x & 0 \le x < 1 \end{cases}$

10%   **8.**   Find product solutions of $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y} + u$.

20%   **9.**   Solve $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2}$, $0 < x < 1$, $t > 0$

Subject to: $u(0,t) = 0$, $u(1,t) = 0$, $t > 0$

$$u(x,0) = 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = \sin \pi x, \quad 0 < x < 1.$$