

Engineering Mathematics

1. (5%) Reduce, by letting $u(x) = [y(x)]^{1-a}$ where a is a constant, to a linear form and solve the following differential equation: (Section 1.6, Prob. 33)

$$y' + \frac{1}{3}y = \frac{1}{3}(1-2x)y'$$

2. (5%) Converting the given equation $y'' - 9y = 0$ to a system $\bar{y}' = \bar{A}\bar{y}$, where \bar{A} is a 2×2 matrix, and then determining \bar{A} . (section 3.1, Prob. 9)

3. (10%) Given the differential equation (Equation (2) on Page 108)

$$y'' + p(x)y' + q(x)y = r(x) \quad (1)$$

with arbitrary variable function $p(x)$, $q(x)$, and $r(x)$ that are continuous on some interval I . Use the method of variation to show a particular solution of (1) on I in the form

$$y_p(x) = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx$$

where y_1 , y_2 form a basis of solutions of the homogeneous equation corresponding to (1), and

$$W = -y_1 y_2' - y_2 y_1'$$

4. (10%) Find a power series solution in powers of x of the following differential equation $y' = -2xy$. (Section 4.2 prob. 1)

5. (10%) Solve the system of equation (section 6.6 Prob. 19)

$$3x + 7y + 8z = -13$$

$$2x + 9z = -5$$

$$-4x + y - 26z = 2$$

by Gramer's rule.

6. (10%) Given $\bar{A} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$. Find the eigenvalues of \bar{A} , and also diagonalization of \bar{A} . (section 6.7 prob. 1)

7. (10%) A vector field is given by $\bar{u} = y^2 \bar{i} + 2xy \bar{j} - z^2 \bar{k}$. Determine the

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divergence of \vec{u} and curl of \vec{u} at the point $(1,2,1)$. Also, determine if the vector field is solenoidal or irrotational.

8. (10%) Use Stoke's theorem to evaluate $\oint_C z^2 \exp(x^2) dx + xy^2 dy + \tan^{-1} y dz$, where C is the circle $x^2 + y^2 = 1$.

9. (a) (5%) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

(b) (5%) By (a), find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

10. (10%) Find the Fourier series of

$$f(x) = \begin{cases} 1 & , -1 < x < 1 \\ x & , 0 \leq x \leq 1 \end{cases}$$

11. (10%) Solve $y \frac{\partial^2 u}{\partial x \partial y} + u = 0$.