1. (10%) Consider the vector function $\vec{F}(x, y, z) = (3x - y)\vec{i} + (y - 4xz)\vec{j} - z\vec{k}$ and the surface S bounding the sphere of radius 4 about $(1,1,1)$. Evaluate $\iint\limits_{S} \vec{F} \cdot \vec{n} dA$.

2. (15%) Solve the linear system: $\begin{cases} x_1 - x_2 + 3x_3 - x_4 = 3 \\ x_2 - 3x_3 + 2x_4 = -2 \\ x_1 + 2x_2 \quad\quad - x_4 = 9 \end{cases}$.

3. Consider the matrix $\mathbf{A} = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$.

(1) (5%) Find the inverse of $\mathbf{A}$.

(2) (10%) A rotation matrix of the form $\mathbf{R} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ can be used to

diagonalize $\mathbf{A}$ to a diagonal matrix $\mathbf{D}$ (*i.e.* $\mathbf{D} = \mathbf{R}'\mathbf{AR}$). Determine $\mathbf{R}$ and $\mathbf{D}$.

4. (15%) Consider the ODE: $(2y^2 - 9xy)dx + (3xy - 6x^2)dy = 0$. Given $y(1) = 1$, solve the initial value problem.

5. (15%) Find the general solution of the ODE: $y''' - 5y'' + 2y' + 8y = 200\sin 2x$.

6. (5%) Find the Laplace Transform of $f(t) = 2t^2 e^{-3t} + 3\sin 6t$.

(A) $\dfrac{4e^{3s}}{s^3} + \dfrac{18}{s^2 + 6}$;　　(B) $\dfrac{4}{(s+3)^3} + \dfrac{18}{s^2 + 36}$;

(C) $\dfrac{4}{(s+3)^2} + \dfrac{3s}{s^2 + 36}$;　　(D) $\dfrac{12}{(s-3)^3} + \dfrac{3}{s^2 - 36}$.

7. Given $\mathcal{F}\{e^{-ax^2}\} = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ax^2} e^{-i\omega x} dx = \dfrac{1}{\sqrt{2a}} e^{-\omega^2/4a}$ for $a > 0$,

(1) (8%) Find the Fourier transform of $f(x) = e^{-(x-3)^2/4}$.

(2) (7%) Find the Fourier transform of $g(x) = (x - 3)e^{-(x-3)^2/4}$.

8. (10%) Given Gamma function, $\Gamma(v) = \int_0^{\infty} e^{-t} t^{v-1} dt$　$(v > 0)$, show that $\Gamma(n+1) = n!$ for integer n.