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＊請在答案卷內作答

本試題均為計算題，應詳列計算過程，無計算過程者不予計分

（一）本大題共有五小題，每小題各占 10 分，共計 50 分。

1. Find the plane through the points $A: \left(1, 2, \frac{1}{4}\right), B: (4, 2, 2)$, and $C: (0, 8, 4)$. (10 分)

2. Prove that the given functions form a basis of the corresponding given equation. Then solve the initial value problem. Show all details of your work. (10 分)

$$e^{-3x}, xe^{-3x}, x^2e^{-3x};$$
$$y''' + 9y'' + 27y' + 27y = 0,\ y(0) = 4,\ y'(0) = -13,\ y''(0) = 46.$$

3. Solve the given equation (10 分)

$$y'' + y = 1 - t^2/\pi^2\ ,\ 0 \le t \le \pi，\ y(0) = 0,\ y'(0) = 0.$$

4. Evaluate

$$\int_C (x^2e^y\vec{i} + y^2e^x\vec{j}) \cdot d\vec{r}$$

counterclockwise around the boundary C of the region R by Green's theorem, where R is the rectangular with vertices $(0, 0), (2, 0), (2, 3), (0, 3)$. Show the details. (10 分)

5. Given

$$\vec{F} = e^x\vec{i} + e^y\vec{j} + e^z\vec{k}.$$

Evaluate the surface integral

$$\iint_S \vec{F} \cdot \vec{n} dA$$

where S is the surface of the cube $|x| \le 1,\ |y| \le 1,\ |z| \le 1$, by the divergence theorem. Show the details. (10 分)

參考用

注意：背面有試題

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（二）本大題共有四小題，共計 50 分。

6. For a general **Legendre** equation $(1-x^2)y''-2xy'+n(n+1)y=0$ Please derive the recursion relation. **(7 分)** and use power series method to solve it as n=1. **(8 分)**

7. Please use **Cramer's rule** to evaluate A_n and B_n of the following equations **(10 分)**

$$\begin{cases} (25-n^2)A_n+0.05nB_n=\dfrac{4}{n^2\pi} \\ -0.05nA_n+(25-n^2)B_n=0 \end{cases}$$

8. $y''+\omega^2 y=r(t); y(0)=K_1; y'(0)=K_2$; Please find the y(t) using Laplace transformation **(15 分)**

9. Solve $y''+2y'+5y=25t-100\delta(t-\pi); y(0)=-2, y'(0)=5$; δ is the direct delta function **(10 分)**

參考用