台灣聯合大學系統 107 學年度學士班轉學考試題

考試科目：工程數學

組別：A5

參考用

―作答注意事項―

1. 作答中如發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。

2. 請核對答案卷（卡）上之准考證號、考試科目是否正確。

3. 本考科可使用電子計算器(一般認知之小型電子計算器)，不限廠牌型號，但不包含手機、平板或其他智慧型手持裝置。

4. 請在答案卷(作答區內)作答。

5. **考生限在作答區內作答，不可書寫姓名、准考證號或與作答無關之其他文字或符號。**

6. 答案卷用盡不得要求增加。

7. 答案卷限用藍筆或黑色鋼筆、原子筆或鉛筆書寫；答案卡限用 2B 軟心鉛筆畫記，如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果考生自行負責。

8. 因字跡潦草或作答未標明題號等情事，致評閱人員無法辨識答案者，該部分不予計分。

（一）本大題共有五小題　　　　　　　　，共計 50 分。。

1. **(10%-- 4%, 6%)** The model of a vibrating elastic string (a violin string, for instance) consists of the one-dimensional wave equation as

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where $u(x,t)$ is lateral vibration displacement, t is time, x is the coordinate along the string, and c=T/ρ (T and ρ denote the tension of the string, and the mass of the undeflected string per unit length, respectively). It's known the string is fastened at the ends $x=0$ and $x=L$; further, the form of the motion of the string will depend on its initial deflection, $f(x)$, and on its initial velocity, $g(t)$.

(i) Write down its boundary conditions and initial conditions.

(ii) Solve the wave equation by separating variables so as to <u>write two ordinary differential equations</u>. [You **do not have to** solve this wave equation thoroughly.]

2. **(10%-- 5%, 5%)** Solve the following linear system of three equations in four unknowns through using **Gauss elimination**.

$$3.0x_1 + 2.0x_2 + 2.0x_3 - 5.0x_4 = 8.0$$
$$0.6x_1 + 1.5x_2 + 1.5x_3 - 5.4x_4 = 2.7$$
$$1.2x_1 - 0.3x_2 - 0.3x_3 + 2.4x_4 = 2.1$$

Please **judge how many solutions** this equation system has and **state the reasons**.

3. **(10%-- 4%, 6%)** For a matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$, the so-called eigenvalue and eigenvector

problem is to solve $\vec{A}\underset{\sim}{x} = \lambda \underset{\sim}{x}$, where λ and $\underset{\sim}{x}$ are the eigenvalue and its corresponding eigenvector.

(i) State the physical meaning of $\vec{A}\underset{\sim}{x} = \lambda \underset{\sim}{x}$.

(ii) Determine the eigenvalues and eigenvectors.

4. **(8%-- 4%, 4%)** Find (i) cur $\vec{v}$ for $\vec{v} = [0, 0, e^{-x}\sin y]$, and (ii) div(grad(fg)) for $f=x+y-z$ and $g=xyz$.

5. **(12%-- 5%, 7%)** Show that the integral $\int_C \vec{F} \cdot d\vec{r} = \int_C (2x\,dx + 2y\,dy + 4z\,dz)$ is path independent in any domain in space and find its value in the integration from the path of

注意:背面有試題

straight line C with two end points A: (0, 0, 0) and B: (2, 2, 2). Please **(i)** first state <u>why</u> the integration is path independent and <u>get the integration result</u>, then **(ii)** get the integration through <u>parametric representation</u> of C.

（二）本大題共有四小題，共計 50 分。

6.　A 12-volt battery is connected to a series circuit in which the inductance is 0.5 henry and the resistance is 10 ohms. Determine the current $i(t)$ if the initial current is zero. **(10 分)**

7. When a cake is removed from an oven, its temperature is measured at 300° F. Three minutes later its temperature is 200° F. How long will it take for the cake to cool off to a room temperature of 70° F? **(15分)**

8. Solve $y'' - 6y' + 9y = t^2 e^{3t}; y(0) = 2, y'(0) = 17$ 。 Hint: use Laplace transform **(10 分)**

9. For a general **Legendre** equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ Please derive the recursion relation. **(7 分)** and use power series method to solve it as n=1. **(8 分)**