台灣聯合大學系統94學年度學士班轉學生考試命題紙

科目：線性代數　類組別：D2　共 1 頁第 1 頁　*請在試卷答案卷（卡）內作答

1. (10 %) Find the inverse of the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ given by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_1\begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix} + x_2\begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix} + x_3\begin{pmatrix} 2 \\ 2 \\ -2 \\ 0 \end{pmatrix} + x_4\begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}.$$

2. (15 %) Show that $W = \{(a_0, a_1, a_2, a_3, a_4) \in \mathbb{R}^5 : a_0 + a_1 + a_2 = 0,\ a_3 - a_4 = 0\}$ forms a subspace of $\mathbb{R}^5$. Find an orthonormal basis for W and an orthonormal basis for $W^{\perp}$.

3. (15 %) Let V, W be two vector spaces with $\dim V < \infty$ and $T : V \to W$ be a linear transformation. Show that $\dim V = \text{nullity}(T) + \text{rank}(T)$.

4. (15 %)

   (a) Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & -2 \\ 0 & -1 & -1 \\ 0 & 0 & -2 \end{pmatrix}$. Are A and B diagonalizable? Justify your answer.

   (b) Are A and B simultaneously diagonalizable? Justify your answer.

   (c) If the answer of (b) is "yes", then find an invertible matrix Q and two diagonal matrices D_1, D_2 such that $Q^{-1}AQ = D_1$ and $Q^{-1}BQ = D_2$. Otherwise skip this question.

5. (15 %) Find $\det A_1$ and $\det A_2$.

$$A_1 = \begin{pmatrix} 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 3 \end{pmatrix}, A_2 = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 3 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 3 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 3 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 3 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}.$$

6. (15 %) Are two matrices $A = \begin{pmatrix} 4 & 6 & -15 \\ 1 & 3 & -5 \\ 1 & 2 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -3 & 3 \\ -2 & -6 & 13 \\ -1 & -4 & 8 \end{pmatrix}$ similar? Justify your answer.

7. (15 %) Sketch the graph of the equation $5x^2 - 6xy + 5y^2 - 24\sqrt{2}x + 8\sqrt{2}y + 56 = 0$.