

台灣聯合大學系統九十二學年度轉學生入學試題卷

類組：A-3 A-4 A-5 年級：2 節次：1 科目：微積分

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甲、填充題：每題 10 分。請將答案依題號寫在答案卷上，不必寫演算過程。

1. Let T be the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$. Find

$$\iint_T e^{\frac{y-x}{y+x}} dA = \underline{\hspace{2cm}}$$

2. Let $A = \int_0^1 e^{-t^2} dt$, $B = \int_0^{\frac{1}{2}} e^{-t^2} dt$. Find $C = \underline{\hspace{2cm}}$ such that

$$\int_{-\frac{1}{2}}^1 \int_0^x e^{-y^2} dy dx = A - \frac{1}{2}B + C.$$

3. Find the maximum value of the function $T(x, y) = x^2 - 2y^2 - x$ on the region $A = \{(x, y) | x^2 + y^2 \leq 1\}$.

4. Let $f(x) = \begin{cases} x^2 - 4x + 6 & \text{if } x \text{ is rational,} \\ x^3 - 6x^2 + 12x - 6 & \text{if } x \text{ is irrational.} \end{cases}$

Find $x_0 = \underline{\hspace{2cm}}$ such that $f'(x_0)$ exists.

5. Let g be a differentiable real-valued function satisfied

$$\left(\int_0^{2x} g(t) dt \right) \sin x + \cos x^2 = x + \exp \left(\int_0^{2x} g(t) dt \right)$$

for all x near 0. Find $g'(0) = \underline{\hspace{2cm}}$.

6. Two holes of radius 6 are drilled through the center of a sphere of radius 10. Assume that their axes meet at right angles. Find the volume of the solid remaining .

7. If $F(x) = \int_1^x f(t) dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u} du$, find $F''(2) = \underline{\hspace{2cm}}$.

乙、計算題：每題 15 分。須詳細寫出演算過程，否則不予計分。

1. Show that the function defined by

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

is not equal to its Maclaurin series.

2. Let a and b be positive numbers with $a > b$. Let a_1 be their arithmetic mean and b_1 their geometics mean: $a_1 = \frac{a+b}{2}$, $b_1 = \sqrt{ab}$. Repeat this process so that, in general, $a_{n+1} = \frac{a_n + b_n}{2}$, $b_{n+1} = \sqrt{a_n b_n}$ for $n \geq 1$. Show that both $\{a_n\}$ and $\{b_n\}$ are convergent and $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n$.

