1. Let v_1, v_2, v_3 be independent vectors in $\mathbf{R}^n$. Let $w_1 = v_1 + 2v_2 + 3v_3$, $w_2 = 4v_1 + 5v_2 + 6v_6$ and $w_3 = 7v_1 + 8v_5$. Show that w_1, w_2 and w_3 are independent. (15%)

2. Find the inverse of $A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 \end{bmatrix}$.

(15%)

3. $A = \begin{bmatrix} 5 & 0 & 0 \\ 3 & 11 & 9 \\ -3 & -6 & -4 \end{bmatrix}$. Find a diagonal matrix D and an orthogonal matrix C such that $C^{-1}AC = D$. (15%)

4. Let $A = \begin{bmatrix} 1 & -1 & 2 & 1 & 0 \\ -1 & 0 & 0 & 1 & 2 \\ 4 & -2 & 0 & 0 & 0 \end{bmatrix}$. Find (a) rank($A$) and (b) det($AA^T$), where A^T is the transpose of A. (15%)

5. Let T be a linear transformation from the vector space V to the vector space W. Let X be a finite dimensional subspace of V and Y is the image $T(X)$ of X. Show that Y is a subspace of W and $\dim(Y) \le \dim(X)$. (15%)

6. Prove or disprove that every $n \times n$ matrix is similar to a diagonal matrix. (10%)

7. Let $b = [1, 1, -1, 2] \in \mathbf{R}^4$ and $W = \{(x,y,z,w) : x+2y-3z+w = 0 \text{ and } 2y+4z-w = 0\}$. Find the projection of b on W. (15%)