國立中央大學九十學年度轉學生入學試題

數學系 三年級 科目：線性代數 共 1 頁 第 1 頁

1. Let $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$, where $\vec{v}_1 = (1, -2, 0, 3, -4)$, $\vec{v}_2 = (3, 2, 8, 1, 4)$, $\vec{v}_3 = (2, 3, 7, 2, 3)$ and $\vec{v}_4 = (-1, 2, 0, 4, -3)$.
(15%) Find a basis for span S.

2. Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $L(x, y, z) = (x+y+z, x+2y+3z)$.
Let $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, $T = \{\vec{w}_1, \vec{w}_2\}$, where $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$
(15%) $\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\vec{w}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{w}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.
Find the matrix of L with respect to S and T.

3. Find an orthogonal matrix P that diagonalizes the matrix
(20%) $A = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}$

4. Let A be an $n \times n$ matrix all of whose entries are integers.
(20%) Show that if $\det(A) = \pm 1$, then all entries of A^{-1} are integers.

5. Suppose that $\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n\}$ is a basis for $\mathbb{R}^n$. Show that if A is an $n \times n$ nonsingular matrix, then
(15%) $\{A\vec{v}_1, A\vec{v}_2, \cdots, A\vec{v}_n\}$ is also a basis for $\mathbb{R}^n$.

6. Let A be an $n \times n$ symmetric matrix. Show that the
(15%) eigenvectors that belong to distinct eigenvalues of A are orthogonal.