國立中央大學八十六學年度轉學生入學試題卷

系 三年級 科目: 高等微積分 共 / 頁 第

1. Assume that $A \subseteq \mathbb{R}^n$ is connected and contain more than one point. Is every point of A an accumulation point of A ? (Explain your answer.) (15%)

2. Each $f_n : \mathbb{R} \mapsto \mathbb{R}$ is uniformly continuous on $\mathbb{R}$, and $\{f_n\}$ converges uniformly to f on $\mathbb{R}$. Is f uniformly continuous on $\mathbb{R}$? (Explain your answer.) (15%)

3. Let $f_n(x) = \sum_{k=1}^{n} \frac{(-1)^k}{k+x}$. Does $\{f_n\}$ converge uniformly on $0 \le x < \infty$? (Explain your answer.) (15%)

4. Evaluate
$$\int_{(0,0)}^{(\pi,2\pi)} (10x^4 - 3x^2y^2)dx - 2x^3y\,dy$$
taken along the path $y = 2x + \sin x$. (15%)

5. Evaluate the integrals
$$\int_1^3 e^{-x}\, d[x] \quad \text{and} \quad \int_0^2 \int_0^2 [x+y]\, dx dy,$$
where $[x]$ is the greatest integer $\le x$. (20%)

6. Show that near $(x_1, x_2, y_1, y_2, y_3) = (0, 1, 3, 2, 7)$ we can solve
$$\begin{cases} 2e^{x_1} + x_2y_1 - 4y_2 + 3 = 0 \\ x_2 \cos x_1 - 6x_1 + 2y_1 - y_3 = 0 \end{cases}$$
uniquely for (x_1, x_2) as functions of (y_1, y_2, y_3) and find the values $\frac{\partial x_1}{\partial y_1}, \frac{\partial x_1}{\partial y_2}, \frac{\partial x_1}{\partial y_3}$ at the point $(y_1, y_2, y_3) = (3, 2, 7)$. (20%)