國立中央大學九十學年度轉學生入學試題

數學系 三年級 科目： 高等微積分 共 壹 頁 第 壹 頁

1. (10%) Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty}[(-1)^n + 2](x-2)^n.$$

2. (15%) Let $w = F(x,y)$ and $y = g(x)$. If F has partial derivatives of order 2 and $g''(x)$ exists, find $\frac{dw}{dx}$ and $\frac{d^2w}{dx^2}$.

3. (15%) Find the line integral

$$\int_C \frac{x}{x^2+y^2}dy - \frac{y}{x^2+y^2}dx$$

along the curve C counterclockwise, where C is the boundary of the square
$S = \{(x,y) : |x| \le 2, |y| \le 2\}$.

4. (20%) Let $f(x,y) = \frac{x^3+y^3}{x^2+y^2}$ for $(x,y) \ne (0,0)$, and $f(0,0) = 0$. Prove that f is continuous at $(0,0)$ and $f_x(0,0)$, $f_y(0,0)$ exist, but f is not differentiable at $(0,0)$.

In Problems 5 and 6, $\mathbb{R}$ denotes the real number system.

5. (20%) Let $a < b$ be real numbers, $f : (a,b) \to \mathbb{R}$. Prove that f is uniformly continuous on (a,b) if and only if f is continuously extendable to $[a,b]$. (Note that f is continuously extendable to [a,b] if there is a continuous function $F : [a,b] \to \mathbb{R}$ such that $F(x) = f(x)$ for all $x \in (a,b)$.)

6. (20%) Let $f_n : \mathbb{R} \to \mathbb{R}$ be nonnegative and continuous on $\mathbb{R}$ for each n. Assume that for each $x \in \mathbb{R}$ the sequence $\{f_n(x)\}$ increases and converges to $f(x)$. If f is continuous on $\mathbb{R}$ and $\lim_{x\to\pm\infty} f(x) = 0$, show that f_n converges to f uniformly on $\mathbb{R}$.