國立中央大學九十一學年度轉學生入學試題卷

物理學系　三年級　　科目：古典物理　　

(25%) 1. Suppose that a particle of mass m moves in the xy-plane under the influence of a force field $\vec{F} = -k(\hat{x}x + \hat{y}y)$, where k is a positive constant, $\hat{x}$ and $\hat{y}$ are unit vectors in the directions of positive x-axis and y-axis of a proper coordinate system, respectively.

(1). Find the equations of motion of the particle.

(2). Show that under the appropriate initial conditions at time $t = 0$, the solutions of the equations of motion are $x(t) = x_0 \cos \omega t$ and $y(t) = y_0 \sin \omega t$, where $\omega = \sqrt{k/m}$, x_0 and y_0 are constants.

(3). Show that the total energy of the particle is $E = \frac{k}{2}(x_0^2 + y_0^2)$.

(4). Show that the orbital angular momentum of the particle with respect to the origin of the coordinate system is $\vec{L} = \hat{x} \times \hat{y} \sqrt{km}\, x_0 y_0$.

(25%) 2. A particle of mass m moves in three dimensions under a central conservative force with potential energy $V(r)$.

(1). Find the Hamiltonian of the particle in terms of spherical polar coordinates (r, θ, ϕ). (Hints: $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$.)

(2). Determine Hamilton's equations of motion of the particle.

(3). Express the quantity $J^2 = m^2 r^4 (\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta)$ of the particle in terms of the generalized momenta and show that it is a constant of motion.

(25%) 3. A very thin ohmic conducting disk of radius b, and conductivity σ_c lies in the xy-plane with the origin at its center. A spatially uniform magnetic field is also present and given by $\vec{B} = \hat{z} B_0 \cos(\omega t + \alpha)$, where B_0, ω, and α are constants. Find the induced current density $\vec{j}$ produced in the disk.

(25%) 4. Consider a linear isotropic homogeneous nonconducting medium of constant permittivity ϵ and constant permeability μ, and with a charge distribution and current distribution of volume charge density $\rho(\vec{r}, t)$ and current density $\vec{j}(\vec{r}, t)$. In the Lorentz gauge, i.e., $\vec{\nabla} \cdot \vec{A} + \mu\epsilon \frac{\partial \Phi}{\partial t} = 0$, find the differential equations satisfied by the vector potential $\vec{A}(\vec{r}, t)$ and the scalar potential $\Phi(\vec{r}, t)$.